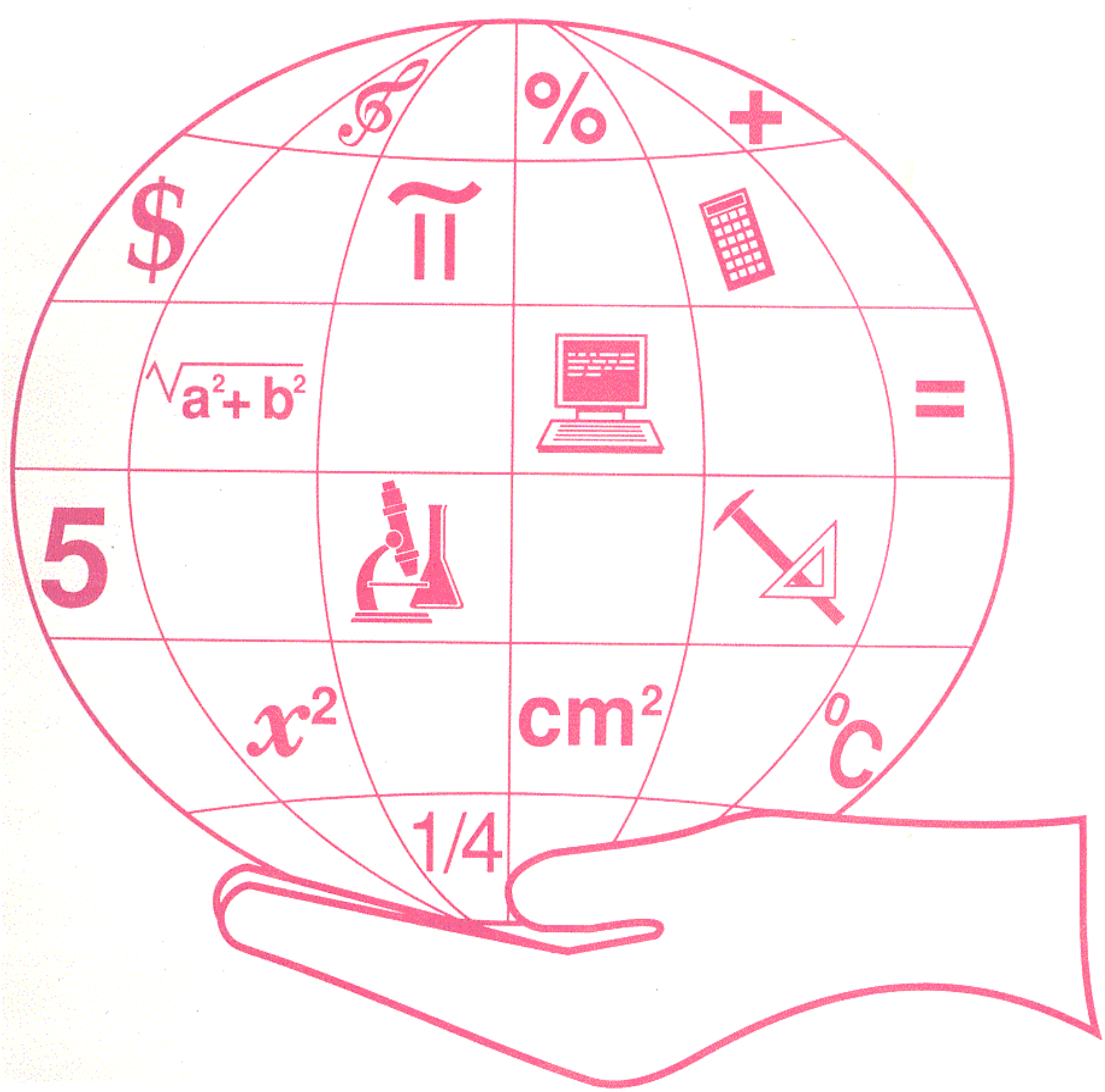
Mathematics 10, 20

A Curriculum Guide for the Secondary Level



September 1995

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Saskatchewan Education,
Training and Employment
September 1995
ISBN 0-921291-17-5

Acknowledgements

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Saskatchewan Education, Training and Employment wishes to thank many others who contributed to the development of the Curriculum Guide:

- pilot teachers;
- the Mathematics Program Team;
- in-house consultants; and,
- various field personnel.

This document was completed under the direction of the Mathematics, Science and Technology Unit at Saskatchewan Education, Training and Employment.

Foreword

Much of the foundation for curriculum renewal in Saskatchewan schools is based on *Directions* (1984). The excitement surrounding the recommendations for Core Curriculum developments will continue to build as curricula are designed and implemented to prepare students for the 21st century.

Mathematics, as one of the Required Areas of Study, incorporates the Common Essential Learnings and the Adaptive Dimension. In addition, other Core Curriculum initiatives such as Gender Equity, Indian and Métis content and perspectives, and Resource-based Learning are also addressed.

As we strive to achieve the goals of mathematics education in Saskatchewan schools, much collaboration and cooperation among individuals and groups will be required. Mathematics teachers are a key part of the process.

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Introduction

Philosophy, Aim, and Goals

The philosophy of mathematics education in Saskatchewan is reflected in the program aim and goals. In addition, the philosophy is closely related to the concept of Core Curriculum based on *Directions* (1984) and the **Goals of Education for Saskatchewan**.

The aim of the mathematics program is to graduate numerate individuals who value mathematics and appreciate its role in society. This is an attempt to enable students to cope confidently and competently with everyday situations that demand the use of mathematical concepts. Specifically, this means interpreting quantitative information, estimating, performing calculations mentally, and developing an intuitive knowledge of measurement and spatial relationships. In addition, the mathematics program is intended to stimulate the spirit of inquiry by developing a variety of problem solving skills and abilities. Lastly, there is a need to make effective use of technology where it is most appropriate.

The curriculum goals are intended to provide for students the mathematical preparation essential to:

- function as consumers and workers; that is, to develop the skills and knowledge of concepts necessary to meet the needs of the average worker and consumer. This can be accomplished by developing an understanding of the relationship between problem solving in the real-world and the mathematics taught in schools;
- function as informed responsible citizens; that is, to develop the ability to analyze and interpret quantitative information.
- obtain a liberal education; that is, to develop logical thinking skills, effective work habits and an appreciation of mathematics;
- become capable problem solvers; that is, to develop the desire, confidence and ability to solve problems;
- communicate mathematically; and,
- pursue further study in mathematics and mathematically related areas.

Emphasis is placed on how to compute, measure, estimate, and interpret mathematical data and when to apply these same skills and techniques. An understanding of why these processes apply is also stressed. The intent is to develop self-reliant, self-motivated, confident life-long learners.

Foundational Objectives

The Foundational Objectives describe the most important understandings and abilities which should be developed over the course of a unit or a year. They provide guidance to teachers in unit and yearly planning and should be within the range of abilities of the majority of students.

The Foundational Objectives form the basis for curriculum evaluation.

The Foundational Objectives are listed at the beginning of the sections dealing with the Mathematics 10 Curriculum and the Mathematics 20 Curriculum. Included with them, for awareness, are the General Outcomes of the Western Protocol, *Common Mathematics Curriculum Framework* (draft, March 1995).

Using the Curriculum Guide

Course Overview

The content of this course is broadly based and practical. The structure of the program includes seven mathematical strands.

i) Data Analysis and Consumer Mathematics
ii) Numbers and Operations
iii) Equations
iv) Algebra
v) Functions
vi) Geometry
vii) Trigonometry

Problem solving is an integral component of all strands, and is to be incorporated throughout the program. **The concept of function is stressed, as the understanding of many of the strands of mathematics depend upon knowledge of this concept.** Concepts are further supported by a number of learning objectives/skills, all of which emphasize and develop the Foundational Objectives of mathematics and the Common Essential Learnings. Mathematics 10 and 20 are intended to meet the needs of all students. These courses are flexible enough to recognize and encourage the development of options to meet the needs of students with both high and low achievement levels.

Any single resource will not cover all of the concepts and skills of this curriculum guide. Instead, a variety of resources/materials should be employed to select activities and content that coincide with student learning styles, individual teaching styles, and the philosophy of the curriculum.

The development and sequencing of the concepts should follow a logical progression using certain necessary principles to show the relationships between the concepts. Teachers should not restrict themselves to using just one instructional strategy when teaching a concept. Teachers might consider the intellectual aptitude of their students, what they already know about the concept, the nature of the concept, its significance in the structure of the other mathematical concepts, and the level of performance expected when choosing among the teaching strategies available.

There are many ways that the Adaptive Dimension can be incorporated into Mathematics 10. They include:

- **altering the method of instruction to meet individual needs;**
- **altering the setting so that students may benefit more fully from instruction;**
- **altering the pace of the lesson to ensure that students understand the concepts; and,**
- **altering the method in which students are required to respond to the teacher and/or to the instructional approach.**

It should be remembered that the less rigid the setting and the approach, the easier it is to adapt. Any method, or some combination of methods, is acceptable. **Suggested ideas are given for helping students to achieve various skills/objectives in this guide in the Instructional Notes and in the Adaptations column.** Additional ideas are provided in *Instructional Approaches: A Framework for Instructional Practice* (Saskatchewan Education, 1991) and *The Adaptive Dimension in Core Curriculum* (Saskatchewan Education, 1992).

Conceptual Overviews

The conceptual overviews for each of Mathematics 10, 20, A30, B30, and C30 are listed in the section ***Aids for Planning***.

The sequencing of these concepts is at the discretion of the teacher. However, a logical order showing the relationships among the various concepts should be considered. For instance, several examples from consumer mathematics can be used to reinforce the concept of linear functions. In order to develop understanding, creative arrangement of the concepts is encouraged. The order in which the concepts are developed could be modified, or several concepts could be integrated.

It should be noted that all the learning objectives for grade 9 have not been included in the Secondary Scope and Sequence. Only those concepts that carry through to successive courses have been indicated.

Reference List for the Scope and Sequence

There are two versions of the "Scope and Sequence" in this guide. One "Scope and Sequence" is a listing of the Foundational Objectives and the supporting learning objectives for each grade, while the second "Scope and Sequence" shows the development of each of the strands over the course of the entire program.

Guide To Using Resource Materials

As was indicated earlier, no single resource matches the Mathematics 10 curriculum. To facilitate a resource-based approach, **the use of a variety of resources instead of a single textbook is highly recommended.**

Teachers may find it necessary for each student to have a basal textbook that covers the majority of the content. If so, then other reference texts must be used. Some teachers may wish to have their students work in groups of two or three and use multiple recommended texts. In any case, the approach should coincide with student learning styles and individual teaching styles. It is also recommended that non-print materials such as software and videos be used in order to enhance the delivery of the course.

A resource-based learning approach requires long-term planning and coordination within a school or school division. In-school administrators, the teacher-librarian, and others need to take an active role to assist with this planning.

Instructional approaches which emphasize group work and develop independent learning abilities make it possible to utilize limited resources in a productive way.

Core Curriculum Components and Initiatives

Core Curriculum: Plans for Implementation (1987) defines the Core Curriculum as including seven Required Areas of Study, the Common Essential Learnings, the Adaptive Dimension, and Locally-Determined Options. Mathematics is one of the Required Areas of Study.

Core Curriculum initiatives also include Gender Equity, Indian and Métis Perspectives, and Resource-based Learning. These initiatives can be viewed as principles which guide the development of curricula as well as instruction in the classroom. The components and initiatives outlined in the following statements have been integrated throughout the curriculum.

Common Essential Learnings

Understanding the Common Essential Learnings: A Handbook for Teachers (1988) defines and expands on an understanding of these essential learnings. These may be considered the "New Basics".

Mathematics offers many opportunities for incorporating the Common Essential Learnings (C.E.L.s) into instruction. The purpose of this incorporation is to help students better understand mathematics and to prepare students for their future learning both within and outside of the K-12 educational system. The decision to focus on particular C.E.L.s within a lesson is guided by the needs and abilities of individual students and by the particular demands of mathematics. Throughout a unit, it is intended that each of the Common Essential Learnings will be developed to the extent possible.

It is important to incorporate the C.E.L.s in an authentic manner. For example, some areas of mathematics may offer many opportunities to develop the understandings, values, skills and processes related to a number of the Common Essential Learnings. The development of a particular C.E.L., however, may be limited by the nature of the subject matter under study.

It is intended that the Common Essential Learnings be developed and evaluated within subject areas. Therefore, Foundational Objectives for the C.E.L.s are included within the overviews of the strands in this guide. Because the Common Essential Learnings are not necessarily separate and discrete categories, it is anticipated that working toward the achievement of one Foundational Objective may contribute to the development of others. For example, many of the processes, skills, understandings and abilities required for the C.E.L.s of Communication, Numeracy, and Critical and Creative Thinking are also needed for the development of Technological Literacy.

Incorporating the Common Essential Learnings into instruction has implications for the assessment of student learning. A unit which has focused on developing the C.E.L.s of Communication and Critical and Creative Thinking should also reflect this focus when assessing student learning. Assessment should allow students to demonstrate their understanding of the important concepts in the unit and how these concepts are related to each other or to previous learning. Questions can be structured so that evidence or reasons must accompany student explanations. If students are encouraged to think critically and creatively throughout a unit, then the assessment for the unit should also require students to think critically and creatively.

It is anticipated that teachers will build from the suggestions in this guide and from their personal reflections in order to incorporate the Common Essential Learnings into mathematics.

For example, involving students in groups to solve realistic problems helps to develop Personal and Social Values and Skills. Similarly, realistic problems provide a medium to promote the important aspects of human communication: listening, speaking, reading and writing. Additionally, critical thinking can be developed in the mathematics program by providing students with an opportunity to evaluate statistical claims made in advertising and by asking "what if" questions in geometry. Numeracy is naturally developed throughout, especially in the interpretation of quantitative information and the use of probability, ratios, and proportions. All serve to assist students to cope confidently and competently with everyday situations. Having students actively using the calculator as a problem solving tool and applying computer spreadsheets to organize data and technological information develops their awareness of technology in an ever changing world. Independent Learning is fostered by encouraging students to investigate the applications, history, and further study of mathematics. In creating such opportunities and experiences, students will become capable, self-reliant, self-motivated, and life-long learners.

Resource-based Learning

Personnel, collections, facilities, and budgets in Saskatchewan school libraries vary a great deal, and the quality of school library programs and services is therefore not consistent. Possibilities for resource-based instruction are related to the level of administrative and staff commitment to developing well-staffed and well-equipped school libraries.

Resource-based teaching and learning is a means by which teachers can greatly assist the development of attitudes and abilities for independent life-long learning. Resource-based learning is student-centred. It offers students opportunities to choose, to explore, and to discover. Students who are encouraged to think critically in an environment rich in resources are well on their way to becoming autonomous learners.

It is important for the mathematics teacher to cooperate with library staff to integrate non-print, human, and print resources with classroom assignments. The classroom teacher plans in advance with library staff, and respects the library resource centre as an extension of the classroom and a place for active learning. The librarian selects materials for the collection based on reviews in professional journals and invites or encourages the input of classroom teachers. The teacher-librarian, if available, could assist with planning assignments, integrating appropriate resources, and teaching students the processes needed to find, use, and present information.

The library resource centre staff could support the mathematics curriculum by:

a) displaying curiosity, and modelling open-ended investigation, and problem-solving approaches;

b) demonstrating use of electronic networks and databases to link to sources that support mathematics interests;

c) organizing and circulating print and non-print resources which support the mathematics curriculum. Resources might include manipulatives, commercial games, videos, filmstrips and films, software, newspapers and magazines, reference books containing statistics and other numerical data, maps and globes, scale drawings, and measuring instruments;

d) maintaining a resource file of speakers and presenters in the community who can contribute their mathematics career experience to the classroom;

e) assisting the mathematics teacher to set up learning stations in the classroom or library using library resources;

f) cooperating with the mathematics teacher to teach students methods of library organization including computerized systems, and practical uses of indexing of all kinds;

g) providing resources for students at all levels of ability including exceptional children;

h) maintaining a collection of professional material on subjects of interest to mathematics teachers;

i) providing a link to information electronic databases and materials from other libraries, the central board office, universities, museums, governments, and industry;

j) providing enrichment materials which anticipate students' interests such as books of puzzles, mathematical games, material on crafts and hobbies using mathematical principles, magazines which deal with mathematics/science, and sports records; and,

k) providing interdisciplinary learning, to help students comprehend and anticipate the links between mathematics and other disciplines and areas of study.

Instructional Approaches

It is necessary for teachers to use a broad range of instructional approaches to give students a chance to develop their understandings and abilities to investigate, to make sense of, and to construct meanings from new situations; to make and provide arguments for conjectures; and to use a flexible set of strategies to solve problems from both within and outside mathematics. In addition, greater opportunities can be provided for small-group work, independent learning, electronic networking, peer instruction, and whole-class discussions in which the teacher serves as a moderator.

Such instructional methods will require **the teacher's role to shift from dispensing information to facilitating learning.** New topics, whenever possible, should be introduced through real-life problem situations that encourage students to explore, formulate and test conjectures, prove generalizations, and discuss and apply the results of their investigations. As a result of such instruction, students should be able to learn mathematics both creatively and independently and thereby strengthen their confidence and skill in doing mathematics. **In fact, problem solving should not only be a means of instruction but also a goal.** The relationship of problem solving to other teaching strategies is very fundamental. One way students can obtain practice in using a problem solving process is for the learning situation to be one where they can discover for themselves the mathematics they are to learn. *Instructional Approaches: A Framework for Professional Practice* (1991) provides additional information to understand and implement a variety of approaches to teaching.

The use of technology in instruction can facilitate the teaching and learning of mathematics. Computer software can be used for class demonstrations and independently by students to explore additional examples, to perform independent investigations, to generate and summarize data as part of a project, or to complete assignments. More information on teaching mathematics using a variety of instructional strategies can be found in *Curriculum Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics, 1989); or from the *Instructional Strategies Series* of booklets published by SIDRU and SPDU.

Adaptive Dimension

The adaptation of instruction to meet learner needs is an expectation inherent in the **Goals of Education** and is an essential ingredient of any consideration of Instructional Approaches. The Adaptive Dimension is defined as:

> "the concept of making adjustments in approved educational programs to accommodate diversity in student learning needs. It includes those practices the teacher undertakes to make curriculum, instruction, and the learning environment meaningful and appropriate for each student." (*The Adaptive Dimension in Core Curriculum*, Saskatchewan Education 1992, page 1)

The continuum of curricular programs authorized by Saskatchewan Education, Training and Employment - Regular, Transitional, and Alternative Programs - recognizes the need for variation in curriculum content and delivery mechanism. As indicated in the continuum, adaptation may be required within each program, and therefore within each course of study. **Teachers are empowered to adjust the curriculum topics and material in order to meet student needs**; as professionals they must ensure that the instructional approaches are also adapted. This implies that teachers have at their "fingertips" a broad, strong repertoire of instructional strategies, methods, and skills and that conscious planning takes place to adapt these approaches to meet student needs.

The cues that some students' needs are not being adequately met come from a variety of sources. They may come to the perceptive teacher as a result of monitoring for comprehension during a lesson. The cue may come from a unit test, or from a student need or background deficiency that has been recognized for several years. A student's demonstrated knowledge of, or interest in, a particular topic may indicate that enrichment is appropriate. The adaptation required may vary from presenting the same content through a slightly different instructional method, to enriching the material because of a known information background deficit or to establishing an individual or small group enrichment activity. The duration of the adaptation may range from five minutes of individual assistance, to provision of an alternative or enrichment program. The diagnosis of the need may be handled adequately by the classroom teacher, or may require the expertise of other support specialists such as the school's resource teacher or other system-based personnel.

The recognition of the need for adaptive instruction is dependent upon the professional judgment of the teacher. The decision to initiate the adaptation may occur through the placement of students in programs other than those defined as regular. The most frequent application of the Adaptive Dimension will occur as teachers in regular classroom settings adjust their use of instructional materials, methods and the environment. Further information can be found in *The Adaptive Dimension in Core Curriculum* (1992).

The flexibility inherent in the Mathematics 10 and 20 curricula accommodates the Adaptive Dimension. Mathematics teachers will have to take advantage of and create inservice opportunities to adjust their repertoire of instructional strategies, methods, and skills.

Gender Equity

Saskatchewan Education, Training and Employment is committed to providing quality education for all students in the K to 12 system. It is recognized that expectations, based primarily on gender, limit students' ability to develop to their fullest potential. Continued efforts are required so that equality of benefit or outcome may be achieved. It is the responsibility of schools to decrease sex-role expectations and attitudes in an effort to create an educational environment free of gender bias. This can be facilitated by increased understanding and use of gender balanced material and strategies, and through further efforts to analyze current practice. Both females and males need encouragement to explore non-traditional as well as traditional options.

In order to meet the goal of gender equity in the K to 12 system, Saskatchewan Education, Training and Employment is committed to efforts to bring about the reduction of gender bias which restricts the participation and choices of students. It is important that Saskatchewan curricula and classrooms reflect the variety of roles and the wide range of behaviours and attitudes available to all members of our society. The new curriculum strives to provide gender balanced content, activities, and teaching strategies described in inclusionary language. These actions will assist teachers to create an environment free of bias and enable both males and females to share in all experiences and opportunities which develop their abilities and talents to the fullest.

Teachers need to believe that both females and males can perform well in mathematics at all grade levels. Teachers should also become aware of the attitudes displayed by their students and help them to view themselves as able to achieve in mathematics. It is important to show students the relevance of mathematics to their own lives, choosing examples which come from the experience of females and males. **From an early age, students need to be made aware that most careers will require mathematics.** Teachers need to be aware of their own interactions with students ensuring that everyone takes an active part. Being aware of interactions between students which may reinforce limiting behaviour or attitudes, and taking opportunities to discuss them, will help students to acquire a broader understanding of their own abilities and potential. All of these actions will support and enhance gender equity in mathematics and move toward improved teaching practice.

Indian and Métis Curriculum Perspectives

The integration of Indian and Métis content and perspectives with the K-12 curriculum fulfils a central recommendation of *Directions, The Five Year Action Plan for Native Curriculum Development* (1984) and *The Indian and Métis Education Policy from Kindergarten to Grade XII* (1989). In general, the policy states:

> Saskatchewan Education recognizes that the Indian and Métis peoples of the province are historically unique peoples and occupy a unique and rightful place in our society today. Saskatchewan Education recognizes that education programs must meet the needs of Indian and Métis

peoples, and that changes to existing programs are also necessary for the benefit of all students. (p. 6)

The inclusion of Indian and Métis perspectives benefits all students in a pluralistic society. Cultural representation in all aspects of the school environment empowers children with a positive group identity. Indian and Métis resources foster a meaningful and culturally identifiable experience for Indian and Métis students, and promote the development of positive attitudes in all students towards Indian and Métis peoples. This awareness of one's own culture and the cultures of others develops self-concept, enhances learning, promotes an appreciation of Canada's pluralistic society, and supports universal human rights.

Saskatchewan Indian and Métis students come from different cultural backgrounds and social environments including northern, rural, and urban areas. Teachers must understand the diversity of the social, cultural, and linguistic backgrounds of Saskatchewan Indian and Métis students. **All educators should be encouraged to learn more about cross-cultural education**, and to increase their awareness of applied sociolinguistics, first and second language acquisition theory, and standard and non-standard usage of language.

Teachers must utilize a variety of teaching strategies that match and build upon the knowledge, cultures, learning styles, and strengths which Indian and Métis students possess. Responsive adaptations to all curricula are necessary for effective implementation.

Saskatchewan teachers are responsible for integrating into the appropriate units of their programs, resources that reflect accurate and sufficient Indian and Métis content and perspectives. Teachers have a responsibility to evaluate all resources for bias and to teach students to recognize such bias.

The following four points summarize the Department's expectations for the appropriate inclusion of Indian and Métis content in curriculum and instruction:

! Curricula and materials will concentrate on positive images of Indian, Métis, and Inuit peoples.

! Curricula and materials will reinforce and complement the beliefs and values of Indian, Métis, and Inuit peoples.

! Curricula and materials will include historical and contemporary issues.

! Curricula and materials will reflect the legal, political, social, economic, and regional diversity of Indian, Métis, and Inuit peoples. (*Indian and Métis Education Policy from Kindergarten to Grade XII*, 1984)

Assessment and Evaluation

Why Consider Assessment and Evaluation?

Much research in education around the world is currently focusing on assessment and evaluation. It has become clear, as more and more research findings accumulate, that a broader range of attributes need to be assessed and evaluated than has been considered in the past. A wide variety of ways of doing this are suggested. Assessment and evaluation are best addressed from the viewpoint of selecting what appears most valid in meeting prescribed needs.

In *Student Evaluation: A Teacher Handbook* (1991) the difference between the various forms of evaluation is explained. Student evaluation focuses on the collection and interpretation of data which would indicate student progress. This, in combination with teacher self-evaluation and program evaluation, provides a full evaluation.

Clarification of Terms

To enhance understanding of the evaluation process it is useful to distinguish between the terms "**assessment**" and "**evaluation**". These terms are often used interchangeably which causes some confusion over their meaning. **Assessment** is a preliminary phase in the evaluation process. In this phase, various strategies are used to gather information about student progress. **Evaluation** is the weighing of assessment information against some standard (such as a curriculum learning objective) in order to make a judgment. Evaluation may then lead to decision and action.

There are three main types of student evaluation: **formative**, **summative**, and **diagnostic evaluation**. Assessment strategies are used to gather information for each type of evaluation.

Formative evaluation is an ongoing classroom process that keeps students and educators informed of students' progress towards program learning objectives. The main purpose of formative evaluation is to improve instruction and student learning. It provides teachers with valuable information upon which instructional modifications can be made. This type of evaluation helps teachers understand the degree to which students are learning the course material and the extent to which their knowledge, understandings, skills, and attitudes are developing. Students are provided direction for future learning and are encouraged to take responsibility for their own progress.

Summative evaluation occurs most often at the end of a unit of study. Its primary purpose is to determine what has been learned over a period of time, to summarize student progress, and to report on progress relative to curriculum objectives to students, parents, and educators.

Seldom are evaluations strictly formative or strictly summative. For example, summative evaluation can be used formatively to assist teachers in making decisions about changes to instructional strategies or other aspects of students' learning programs. Similarly, formative evaluation may be used to assist teachers in making summative judgments about student progress. However, it is important that teachers make clear to students the purpose of assessments and whether they will later be used summatively.

Diagnostic evaluation usually occurs at the beginning of the school year or before a unit of instruction. Its main purposes are to identify students who lack prerequisite knowledge, understanding, or skills, so that remedial help can be arranged; to identify gifted learners to ensure they are being sufficiently challenged; and to identify student interests. Diagnostic evaluation provides information essential to teachers in designing appropriate programs for students.

It is typical to conduct all three types of evaluation during the course of the school year.

Phases of the Evaluation Process

Although **evaluation** is not strictly sequential, it can be viewed **as a cyclical process including four phases: preparation, assessment, evaluation, and reflection.** The evaluation process involves the teacher as a decision maker throughout all four phases.

- In the **preparation phase**, decisions are made which identify what is to be evaluated, the type of evaluation (formative, summative, or diagnostic) to be used, the criteria against which student learning outcomes will be judged, and the most appropriate assessment strategies with which to gather information on student progress. The teacher's decisions in this phase form the basis for the remaining phases.

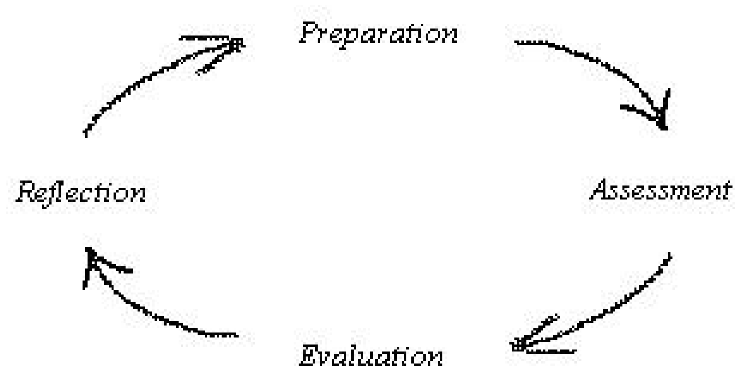
- During the **assessment phase**, the teacher identifies information-gathering strategies, collects student products, constructs or selects instruments, administers them to the student, and collects the information on student learning progress. The teacher continues to make decisions in this phase. The identification and elimination of bias (such as gender and culture bias) from the assessment strategies and instruments, and determining where, when, and how assessments will be conducted are examples of important considerations for the teacher in this phase of evaluation.

- During the **evaluation phase**, the teacher interprets the assessment information and makes judgments about student progress. Based on the judgments or evaluations, teachers make decisions about student learning programs and report on progress to students, parents, and appropriate school personnel.

- The **reflection phase** allows the teacher to consider the extent to which the previous phases in the evaluation process have been successful. Specifically, the teacher evaluates the utility and appropriateness of the assessment strategies used. Such reflection assists the teacher in making decisions concerning improvements or modifications to subsequent teaching and evaluation.

All four phases are included in formative, diagnostic, and summative evaluation processes. They are represented in Figure 1.

Figure 1. Process of Student Evaluation



Guiding Principles

Nine guiding principles are presented in the final report of the Minister's Advisory Committee on Evaluation and Monitoring, entitled, *Evaluation in Education* (1989). The purpose of these principles is to provide guidance on educational evaluation in several areas. One of these areas is student evaluation. The evaluation of student progress has a strong influence on both teaching and learning. If used appropriately, evaluation can promote learning, build confidence, and develop students' understanding of themselves.

Five general guiding principles provide a framework to assist teachers in planning for student evaluation:

1. Evaluation is an essential part of the teaching-learning process. It **should be a planned, continuous activity** which is closely linked to both curriculum and instruction.

2. Evaluation should be **guided by the intended learning outcomes of the curriculum**, and **a variety of assessment strategies should be used**.

3. Evaluation plans **should be communicated in advance**. Students should have opportunities for input to the evaluation process.

4. Evaluation should be **fair and equitable**. It should be sensitive to family, classroom, school, and community situations; it should be free of bias. Students should be given opportunities to demonstrate the extent of their knowledge, understandings, skills, and attitudes.

5. Evaluation **should help students.** It should provide positive feedback and encourage students to participate actively in their own learning.

Assessing Student Progress

Specific assessment strategies are selected or devised to gather information related to how well students are achieving the learning objectives of the curriculum. The assessment strategies used at any given time will depend on several factors such as the type of learning outcomes (knowledge, understanding, skill, attitude, value, or process), the subject area content, the instructional strategies used, the student's level of development, and the specific purpose of the evaluation.

Various assessment strategies are listed in Table 1 as a reference for teachers. The assessment strategies are not prescribed; rather, they are meant to serve only as suggestions, as the teacher must exercise professional judgment in determining which strategies suit the specific purpose of the evaluation. **It would be inappropriate for curriculum guides to give teachers specific formulas for assessing students.** Planning for assessment and evaluation must take into account unique circumstances and purposes which will vary. For further information on the various assessment strategies and types of instruments that can be used to collect and record information about student learning, refer to *Student Evaluation: A Teacher Handbook* (1991).

Common Essential Learnings (C.E.L.s) will be incorporated in the Foundational Objectives of each course. As each subject area is assessed and judgments are made, the C.E.L.s will form an integral part of the evaluation process within the area of study. For example, in a unit of instruction, some learning objectives will identify expected learning outcomes associated with C.E.L.s, but they will be embedded within the subject area content. Assessment strategies will be used to gather student progress information on C.E.L.s through the subject area. When all assessment information has been gathered, it will form the basis for an evaluation. **It is inappropriate to evaluate student progress in the Common Essential Learnings independent of the subject area content.**

A Reference List of Assessment Strategies

Table 1. **Assessment Strategies**

Student Classroom Performance
Anecdotal Records

Observation Checklist

Rating Scale

Contract

Laboratory Report

Portfolio

Test Station

Peer and Self-Assessment

Major Projects, Written Reports

Discussion Groups

Teacher Developed Test Formats
Matching Item

Multiple Choice

Oral

Short Answer

True/False

Essay

Performance Test

Student Assessment in Mathematics

At the beginning of any course, the Foundational Objectives and the learning objectives for the curriculum become the criteria to assess the student. These objectives may be attainable by the majority of students, but for some students, these objectives may not be attainable. Adaptations to instruction or procedures may be required.

A teacher must be aware that "graded" teaching resources and standardized tests are built on what is accepted as average for a student of a given age group and segment of society. In using standardized test, a teacher is assessing how a student matches these cultural standards over a very narrow range of skills. The results must be considered in that context. These standards may be unattainable by some students. Alternatively, some students may not reach full potential because they are not challenged but are allowed to remain at the acceptable "average". The Adaptive Dimension recognizes that the needs of all students must be considered for effective teaching and learning to occur.

The learning of mathematics is a cumulative process that occurs as experiences contribute to understanding. A numerical grade offers only a glimpse of a student's knowledge. If the goal of assessment is to obtain a valid and reliable picture of a student's understanding and achievement, evidence must come from a variety of sources. These sources may include oral presentations, written work, observations, or various combinations of these. Examples of written work include projects, homework assignments, journals, essays, quizzes, and exams. Records of a student's progress may include anecdotal records, portfolios, and mathematical journals. Rating scales and observation checklists are also helpful devices to record evidence of a student's continued growth in understanding. The advantage of using several kinds of assessments is that a student's understanding can be continuously monitored. In addition, because students differ in their perceptions and thinking styles, it is crucial that they are given the opportunity to demonstrate their individual capabilities. A single type of assessment can frustrate students, diminish their self-confidence, and make them feel anxious about mathematics. Examples of various templates for assessment and evaluation are included in this guide.

The assessment of a student's mathematical knowledge includes the ability to solve problems, to use the language of mathematics, to reason and analyze, to comprehend the key concepts and procedures, and to think and act in positive ways. Assessment should also examine the extent to which students have integrated and made sense of mathematical concepts and procedures and whether they can apply these concepts and procedures to situations that require creative and critical thinking.

Understanding concepts and their interrelationships is essential to interpreting a situation and deriving an appropriate plan of action. Knowing what procedures are appropriate and how to execute them is essential to carrying out the plan successfully.

Methods for assessing a student's ability to solve problems include observing the student solving problems individually, in small groups, or in class discussions. Other methods include listening to a student discuss problem-solving processes and analyzing tests, homework, journals, and essays. A rating scale is useful for assessing a student's problem solving skills. It may include rating a student's willingness to engage in problem solving, the use of a variety of strategies, facility in finding the solution to problems, and consistency in verifying the solution.

Assessment of a student's ability to communicate mathematically includes the meaning he/she attaches to the concepts and procedures of mathematics. It also involves his/her ability in talking about, writing about, understanding, and evaluating mathematical ideas. In assessing a student's ability to communicate, attention should be given to the clarity, precision, and appropriateness of mathematical terms and symbols. Discussion is also a splendid means of judging a student's ability to function as a critical participant in small groups or within the class.

The assessment of a student's ability to communicate through the use of computers and software, such as spreadsheets and data-base and function plotting programs, is also important. A student's ability to structure and present information with the use of technology can be assessed by determining if a student can use a spreadsheet or graph to simulate a situation or provide evidence for a conclusion.

An understanding of mathematical concepts involves more than mere recall of definitions and recognition of examples. It also encompasses a broad range of abilities. Assessment must include the aspects of conceptual understanding by focusing on a student's ability to discriminate between relevant and irrelevant attributes of a concept in selecting examples and non-examples, to represent concepts in various ways, and to recognize their various meanings. Observational checklists, anecdotal records or written reports may be used to assess such conceptual understanding.

Learning mathematics also includes developing a positive attitude towards mathematics. The assessment of a student's attitude requires information about her/his thinking and actions in a wide variety of situations. A student's attitudes are reflected in how he/she asks and answers questions, works on problems, and approaches new mathematics. Observations, homework assignments, journals, and oral presentations are all excellent ways to assess a student's mathematical attitude.

Program Evaluation

Program evaluation is a systematic process of gathering and analyzing information about some aspect of a school program in order to make a decision, or to communicate to others involved in the decision-making process. Program evaluation can be conducted in two ways: relatively informally at the classroom level, or more formally at the classroom, school, or school division levels.

At the classroom level, program evaluation is used to determine whether the program being presented to the students is meeting both their needs and the objectives prescribed by the province. Program evaluation is not necessarily conducted at the end of the program, but is an ongoing process. For example, if particular lessons appear to be poorly received by students, or if they do not seem to demonstrate the intended learning from a unit of study, the problem should be investigated and changes made. **By evaluating their programs at the classroom level, teachers become reflective practitioners.** The information gathered through program evaluation can assist teachers in program planning and in making decisions for improvement. Most program evaluations at the classroom level are relatively informal, but they should be done systematically. Such evaluations should include identification of the area of concern, collection and analysis of information, and judgment or decision making.

Formal program evaluation projects use a step-by-step problem-solving approach to identify the purpose of the evaluation, draft a proposal, collect and analyze information, and report the evaluation results. The initiative to conduct a formal program evaluation may originate from an individual teacher, a group of teachers, the principal, a staff committee, an entire staff, or central office. Evaluations are usually done by a team, so that a variety of skills are available and the work can be distributed. Formal program evaluations should be undertaken regularly to ensure programs are current.

To support formal school-based program evaluation activities, the Department has developed the *Saskatchewan School-Based Program Evaluation Resource Book* (1989) to be used in conjunction with an inservice package.

Curriculum Evaluation

During the decade of the 1990s, new curricula are being developed and implemented in Saskatchewan. Consequently, there is a need to know whether these new curricula are being effectively implemented and whether they are meeting the needs of students. Curriculum evaluation, at the provincial level, involves making judgments about the effectiveness of provincially authorized curricula.

Curriculum evaluation involves gathering information (the assessment phase) and making judgments or decisions based on the information collected (the evaluation phase), to determine how well the curriculum is performing. **The principal reason for curriculum evaluation is to plan improvements to the curriculum.** Such improvements might involve changes to the curriculum document and/or the provision of resources or inservice to teachers.

It is intended that curriculum evaluation be a shared, collaborative effort involving all of the major education partners in the province. Although the Department is responsible for conducting curriculum evaluations, various agencies and educational groups are involved. For instance, contractors are hired to design assessment instruments; teachers are involved in instrument development, validation, field testing, scoring, and data interpretation; and the cooperation of school divisions and school boards is necessary for the successful operation of the program.

In the assessment phase, information is gathered from students, teachers, and administrators. The information obtained from educators indicates the degree to which the curriculum is being implemented, the strengths and weaknesses of the curriculum, and the problems encountered in teaching it. The information from students indicates how well they are achieving the intended learning outcomes and provides indications about their attitudes toward the curriculum. Student information is gathered through the use of a variety of strategies including paper-and-pencil tests (objective and open-response), performance (hands-on) tests, interviews, surveys, and observation.

As part of the evaluation phase, assessment information is interpreted by representatives of all major education partners including the Department and classroom teachers. The information collected during the assessment phase is examined, and recommendations, generated by an interpretation panel, attempt to address areas in which improvements can be made. These recommendations are forwarded to the appropriate groups such as the Curriculum and Instruction Branch, school divisions and schools, universities, and educational organizations in the province.

All provincial curricula will be included within the scope of curriculum evaluation. Evaluations will be conducted during the implementation phase for new curricula, and regularly on a rotating basis thereafter. Curriculum evaluation is described in greater detail in *Curriculum Evaluation in Saskatchewan* (1990).

Teacher Self-Evaluation

There are two levels of teacher self-evaluation: reflection on day-to-day classroom instruction and professional self-evaluation.

Teachers refine their skills through reflecting upon elements of their instruction which includes evaluation. The following questions may assist teachers in reflecting on their evaluations of student progress:

- Was there sufficient probing of student knowledge, understanding, skills, attitudes, and processes?
- Were the assessment strategies appropriate for the student information required?
- Were the assessment conditions conducive to the best possible student performance?
- Were the assessment strategies fair/appropriate for the levels of student abilities?
- Was the range of information collected from students sufficient to make interpretations and evaluate progress?
- Were the results of the evaluation meaningfully reported to students, parents, and other educators as appropriate?

Through reflection on questions like those above, teachers are able to improve their strategies for student evaluation.

It is also important for teachers, as professionals, to engage in self-evaluation. Teachers should take stock of their professional capabilities, set improvement targets, and participate in professional development activities. Some ways teachers can address their professional growth are by: reflecting on their own teaching; reading professional documents (e.g., articles, journals and books); attending workshops, professional conferences, and courses; and developing networks with other professionals in their fields.

Information Gathering and Record-Keeping

Having summarized the various types of assessment and evaluation, it is obvious that large amounts of data are gathered by teachers, schools and school divisions, and the Department. It is **important that teachers maintain appropriate records** to ensure data are organized and accessible for making judgments and decisions. Records can be kept in a variety of ways; however, it is **recommended that teachers keep separate files on student progress** (student portfolios), **teachers' self-evaluations** (professional files), and **program evaluation**. Schools and school divisions also keep records of student enrollment and progress which support decision making at the local level. Saskatchewan Education, Training and Employment is developing and implementing a comprehensive student record system with the capacity to register students K-12. This database will assist schools, school divisions, and the province to make informed decisions related to areas such as student mobility, dropout rates, retention rates, and student ethnicity. Saskatchewan Education, Training and Employment also requires comprehensive information to make informed decisions at the provincial level in areas such as program and curriculum evaluation.

Conclusion

Evaluation is the reflective link between what ought to be and what is, and therefore, it is an essential part of the educational process. The main purposes for evaluating are to facilitate student learning and to improve instruction. By continuously evaluating student progress, school programs, curriculum, and the effectiveness of instruction and evaluation, these purposes will be realized.

Program Organization

Conceptual Teaching

Concepts are the basis of formal education. Since formal education proceeds mainly through language and is highly concentrated, concepts are of great importance. Principles, generalizations, and rules of procedures are also formulated by means of concepts. Attitudes too, are grounded on concepts. Consequently, it is through the understanding of the fundamental mathematical concepts and their meaning or interpretation that students will make sense of mathematics.

One way an individual can respond to collections of objects is by distinguishing among them. Another way, even more important as a human capability, is by putting things into a class and responding to the class as a whole. The latter type of learning, which makes it possible for the individual to respond to things or events as a class, is called Concept Learning.

Concept teaching lends itself well to mathematics because it is process oriented. Concept teaching approaches are aimed at teaching students to think, to question and to discover rather than to memorize. A conceptual approach also encourages inductive thinking as students move from particular facts to generalizations. It enables students to sort more effectively through the multitude of mathematical information by stressing the commonality among this information. Students are provided with the opportunity to examine and experience and to develop a basis for enhancing their understanding of the concepts they form and acquire. This approach to learning encourages the development of Critical and Creative Thinking and promotes Independent Learning.

There are various strategies that are effective in teaching concepts. However, the intellectual aptitude of the student, what he/she may already know about the concept, the nature of the concept, its significance in the structure of other mathematical concepts, and the level of performance expected are all factors that a teacher should consider in choosing the most appropriate strategy. As a result, the student will develop the abilities to discriminate between relevant and irrelevant attributes of a concept in selecting examples and non-examples, to represent concepts in various ways, and to recognize their meanings.

Integration

Course content should be presented within the context of its application in daily living, integrated within the various branches of mathematics, and related to other academic disciplines. Integration may include any one of the several forms.

- Interdisciplinary Studies - combining subjects
- Thematic - by topic or by concepts
- Holistic Approach - giving the big picture first
- Infusion - integrating technologies or teaching strategies, into the school program
- Integrated Brain work - allows for seeking and creating meaningful organization by the individual
- Using All Mind/Brain Functions in Learning - cognitive, affective, physical/sensing, and intuitive

Teachers need to be familiar with the mathematical competence required of students in the particular course of study, everyday life, and other academic disciplines. Cooperative planning and conferencing with other teachers is central to understanding differing contexts in which basic mathematical skills are used, and will assist teachers in providing practical learning experiences that encourage transfer of knowledge and skill.

Problem Solving

Problem solving plays an integral role in the mathematics program so that students are provided with some of the thinking and problem solving skills necessary to help explain the world around them. Problem solving is the process of accepting a challenge and striving to resolve it. It allows students to become skillful in selecting and identifying relevant conditions and concepts, searching for appropriate generalizations, formulating plans, and employing acquired skills.

In 1945, George **Polya** published the book *How To Solve It* in which he outlined a **four-step model** which could be used during the solution of problems. It involves **understanding the problem, devising a plan, carrying out the plan, and looking back.** Problem solvers can learn individual skills and strategies to use within this framework. As they broaden their knowledge, their facility for solving problems will improve. Polya's framework is not fixed. Although problem solvers may approach a solution in the order outlined, they will often return to earlier stages because they encounter an obstacle and it becomes obvious that another approach will work better.

Problem solving is a process which is learned by doing. Students will become better problem solvers if they think that the activity is important and relevant. This importance is enhanced by observing their teacher(s) solving problems and by their teacher(s) expecting them to do the same. Teaching the four-step problem solving model lends itself to incorporating the Common Essential Learnings; in particular, Critical and Creative Thinking.

When mathematical concepts and operations are introduced, they should often follow rather than precede problem-solving opportunities. Before the formal terms and symbols are presented, students should learn to approach problems in a variety of ways. The problems should consist of a good mix of process, realistic, and translation problems.

Process problems are those that generally can not be solved using routine procedures; rather, their solution typically involves the application of some problem solving strategy or heuristic. **Realistic problems** are not well defined and require some further specification and refinement, often have multiple solutions, require the collection of information, involve collaboration with other people, cannot be solved in a few minutes, and involve some personal commitment on the part of the student. **Translation** problems involve translating written or verbal statements into mathematical expressions and then performing an algorithm.

Some of the problem solving strategies that students should develop include:

- representing the problem situation with an appropriate diagram, model, or simulation;
- incorporating the data into an organized list, table or chart, suitable graph, or tree diagram;
- guessing and checking, substituting a guessed number, carrying out a calculation, checking the suitability of the answer, making an adjustment and repeating the process;
- changing a point of view;
- reconstructing and solving a related, but more simple problem;
- looking for a numerical or geometrical pattern which can be generalized;
- using logical thinking to eliminate possibilities; and,
- working backwards.

Estimation

Estimation is an important mathematical and life skill necessary for effective problem solving, calculations, and calculator use. The desired accuracy of an estimation depends entirely on its use. Whether to produce a rough estimate, fine estimate, or an exact answer depends on the ultimate use of the estimate. Most often practical situations involve estimations rather than exact numbers. It should also be remembered that estimation produces answers that are not exact, but that are adequate for making necessary decisions.

Mental Calculations

Mental calculation is an important and frequently used practical life skill. Despite the increasing advances in calculator technology, mental calculations will remain a convenient calculation tool for judging a narrow range of quantitative problems. It can also improve the efficiency of pencil and paper calculations by reducing the number of steps needed to work out a written calculation. The close relationship between estimating and calculating mentally implies that teachers cannot ignore instruction on mental calculations. After all, mental calculation is the cornerstone for all estimation procedures.

To assist students in developing the ability to calculate mentally it is beneficial to:

- encourage students to calculate mentally whenever possible;
- expect students to explain their methods whenever possible;
- practise mental calculations in a meaningful setting;
- treat mental calculations as a skill to be used in all areas of the program;
- not discourage capable students from skipping steps when performing written calculations;
- provide regular practice and review in learning the basic number facts;

- allow students to develop their own informal approaches to calculate and solve problems; and,
- help students recognize that there is no best method of calculating mentally.

Calculators and Computers

Through the course of time, humans have tried to develop computing devices to provide easier and more accurate computations. The calculator and computer have become essential tools in both business and industry. They have also moved into the home with many families owning a calculator or a computer.

In mathematics, calculators are useful tools in order to:

- develop concepts;
- explore relationships;
- explore patterns;
- organize and display data;
- eliminate tedious computations;
- encourage students to be inquisitive and creative;
- develop wise consumers;
- reinforce the learning of basic number facts and properties;
- develop understanding of computational algorithms;
- promote independence in problem solving;
- decrease the time spent on computations; and,
- promote Numeracy.

The National Council of Teachers of Mathematics (NCTM) recommends that in the classroom, the calculator should be used in imaginative ways to reinforce learning and to motivate learners as they become proficient in mathematics. To use calculators effectively, students must be able to judge the reasonableness of an answer and understand the importance of making a judgment about the results of a calculation.

Interactive computer software holds great promise for application in the mathematics classroom. Its value in creating geometric displays, organizing and graphing data, simulating real-life situations, demonstrating mathematical relationships, and generating numerical sequences and patterns is evident.

The NCTM *Curriculum and Evaluation Standards for School Mathematics* (1989) suggests that:

- all students should have a calculator, possibly one that has graphing capabilities;
- a computer should be available at all times in every classroom for demonstration purposes; and,
- all students should have access to computers for individual and group work.

Manipulatives

Manipulative materials provide students with a concrete base upon which to build concepts and skills. **Students who experience difficulty with mathematical processes should** be given opportunities to develop an understanding of number operations and relationships through tactile and visual learning activities. Students also need to observe, question, verbalize, and discuss the relationships at the concrete level, and eventually to translate relationships into abstract symbols of mathematics. While the purpose of manipulatives is to help students understand and remember, each student should become efficient in making application of concepts in their abstract form. It is important, therefore, that secondary teachers supplement and alternate the use of manipulative materials with other more abstract learning strategies and activities.

Aids for Planning

! Scope and Sequence (by course)
! Scope and Sequence (by strand)

Scope and Sequence for Secondary Mathematics (by course)

Mathematics 10

Major Concepts	Number of Hours
A. Linear Equations and Inequalities	12
B. Relations, Linear Functions, and Variation Part I: Theory ! Linear Functions ! Linear Equations ! Slope Part II: Applications ! Direct Variation ! Partial Variation ! Applications - Arithmetic Sequences	28
C. Consumer Mathematics ! Income ! Budget	10
D. Lines and Line Segments ! Parallel Lines ! Perpendicular Lines	5
E. Angles and Polygons ! n-gons ! Triangles ! Quadrilaterals ! Parallelograms	10-15
F. Review of Algebraic Skills ! Numbers and Operations ! Exponents ! Polynomials	5-10
G. Optional Topics ! Direction Vectors a) on a Number Line b) in a Plane ! Surveying ! Integration with other subject areas	
Total	70-80

! The remaining available time may be spent on additional practice, enrichment, or extension of the course.

In this scope and sequence, each of the Foundational Objectives will be supported by the learning objectives numbered in each case. The numbering in parentheses after each Foundational Objective is a coding that could be employed in any specific assessment.

Objectives

A. Linear Equations and Inequalities

Foundational Objective

- To demonstrate the ability to solve linear equations and inequalities. (10 01 01). Supported by the following learning objectives:

1. To solve and verify the following types of equations in one variable by applying formal algebraic rules: equations containing variables on both sides of the equal sign, equations containing parentheses, equations containing fraction or decimal coefficients.

2. To solve a formula for an indicated variable.

3. To solve, graph, and verify linear inequalities in one variable.

4. To translate English phrases into mathematical terms and vice versa.

5. To solve real-world problems using various problem-solving strategies.

B. Relations, Linear Functions, and Variation

Foundational Objectives

- To be aware that the graph of a first degree equation of two variables is a line, and conversely that the equation of a line is a first degree equation of two variables. (10 02 01). Supported by learning objectives 1, 2, 3, 4, 5, and 9.

- To produce graphs of linear equations, and conversely, when given key information of the graph, find its equation. (10 02 02). Supported by learning objectives 6, 7, 8, 10, 13, 14, 15, 16, 17, and 18.

- To use the knowledge of linear functions and equations to solve problems involving direct and partial variation. (10 02 03). Supported by learning objectives 19 to 23.

- To use the concept of slope to measure the steepness of a line. (10 02 04). Supported by learning objectives 11 and 12.

- To demonstrate the ability to work with arithmetic sequences. (10 02 05). Supported by learning objectives 24 to 27.

Part I: Theory

1. To define the following terms: relation, ordered pair, abscissa, ordinate, function, linear function, slope, x-intercept, y-intercept, ratio, proportion, direct variation, partial variation.

2. To identify and express examples of relations in the real world.

3. To graph ordered pairs in the Cartesian coordinate plane.

4. To graph real-world relations in the Cartesian coordinate plane.

5. To read information from a graph.

6. To identify, graph, and interpret examples of linear functions describing real-world situations.

7. To graph a linear function using a table of values.

8. To determine if a relation is a function by employing the vertical line test.

9. To solve equations in two variables given the domain of one of the variables.

10. To determine if an ordered pair is a solution to the linear equation.

11. To calculate the slope of a line graphically and algebraically when given two points that lie on the line.

12. To determine the slope of horizontal lines, vertical lines, parallel lines, and perpendicular lines.

13. To graph a linear equation using the x- and y- intercepts.

14. To graph a linear equation using the slope and y-intercept.

15. To write the equation of a line when given the slope and one point on the line.

16. To write the equation of a line when given two points that lie on the line.

17. To construct scatterplots from real-world data.

18. To interpret and critically analyze these constructed scatterplots.

Part II: Applications

19. To identify, describe, and interpret examples of direct variation in real-world situations.

20. To solve proportions involving direct variation.

21. To solve problems involving direct variation.

22. To identify partial variation.

23. To solve problems involving partial variation.

24. To define, illustrate, and identify an arithmetic sequence.

25. To determine the n^{th} term of an arithmetic sequence.

26. To define arithmetic means, and to determine the required arithmetic means between given terms.

27. To calculate the sum of an arithmetic series.

C. Consumer Mathematics

Foundational Objectives

! To apply simple mathematics to assist in the calculation and estimation of income and expenses and to develop a budget to guide current and future planning. (10 03 01). Supported by learning objectives 1 to 9.

! To communicate a summary of financial projections in appropriate reports, tables, and graphs. (10 03 02). Supported by learning objectives 10 to 12.

1. To calculate weekly gross wages involving regular pay, overtime pay, and piecework earnings.

2. To calculate earnings for straight commission, or base wage plus commission.

3. To determine the difference between gross pay and net pay.

4. To calculate weekly, monthly, and yearly net pay.

5. To define and explain the purpose of a budget.

6. To determine and calculate monthly fixed expenditures.

7. To investigate the guidelines in developing a budget.

8. To plan a budget based on percentages allotted to various categories as suggested by financial institutions.

9. To calculate the portion of total income spent on each category using percents.

10. To draw graphs (including circle graphs) of budget figures, using appropriate software.

11. To calculate the actual amount of money to be spent on each category using the predetermined percentages.

12. To adjust a budget to changes in expenses.

D. Lines and Line Segments

Foundational Objective

! To develop an informal understanding of the relationships between lines. (10 04 01). Supported by the following learning objectives.

1. To define line segment, ray, line, bisector, perpendicular line, perpendicular bisector, transversal, alternate interior angles, corresponding angles, same-side interior angles.

2. To identify and calculate the measures of the following angles formed by parallel lines: corresponding angles, alternate interior angles, and same-side interior angles.

3. To solve word problems involving angles formed by parallel lines.

4. To informally construct a line parallel to a given line through a point not on the line.

5. To informally construct a line perpendicular to a given line from a point on the line.

6. To informally construct a line perpendicular to a given line through a point not on the line.

7. To informally construct the perpendicular bisector of a line segment.

Informally construct means employing the mira or paperfolding. Traditional straight edge and compass constructions may be used as enrichment.

E. Angles and Polygons

Foundational Objectives

! To identify and apply common properties of triangles, special quadrilaterals, and n-gons. (10 05 01). Supported by learning objectives 1, 2, 3, 4, 5, 8, 9, and 10.

! To apply the special quadrilaterals to real-world situations. (10 05 02). Supported by learning objectives 6 and 7.

! To develop an understanding of Pythagoras' Theorem, the primary trigonometric ratios, and their applications. (10 05 03). Supported by learning objectives 11 to 17.

1. To define and illustrate by drawing the following: acute angle, right angle, obtuse angle, straight angle, reflex angle, complementary angles, supplementary angles, adjacent angles, vertically opposite angles, congruent angles, central angles of a regular polygon.

2. To solve word problems involving the angles stated in #1.

3. To define and illustrate the following polygons: convex, non-convex, regular, quadrilateral parallelogram, rectangle, rhombus, square, trapezoid, and isosceles trapezoid.

4. To classify quadrilaterals as trapezoids, isosceles trapezoids, parallelograms, rectangles, rhombuses, and squares.

5. To informally construct parallelograms, rectangles, rhombuses, and squares.

6. To state and apply the properties of parallelograms.

7. To determine the sum of the measures of the interior and exterior angles of a convex polygon of n sides.

8. To determine the measure of a central angle in a regular n-gon.

9. To determine the measures of the interior and exterior angles of regular n-gons.

10. To determine the number of diagonals in a polygon of n sides.

11. To calculate to two decimal places the length of a missing side of a right triangle using the Pythagorean Theorem.

12. To solve word problems using the Pythagorean Theorem.

13. To determine if a triangle is a right triangle by using the converse of the Pythagorean Theorem.

14. To determine the value of the three primary trigonometric ratios by using a calculator.

15. To determine the measure of an angle given the value of one trigonometric ratio of the angle by using a calculator.

16. To calculate the measure of an angle or the length of a side of a right triangle using the tangent, sine, and cosine ratios.

17. To solve word problems that involve trigonometric ratios using a calculator.

F. Review of Algebraic Skills

This is a review of grade 9 and is intended to precede Mathematics 20. Some of these objectives have not been expanded in this guide.

Foundational Objective

! To make the transition from arithmetic skills to algebraic skills. (10 06 01). Supported by the following learning objectives.

1. Numbers and Operations
   a) To represent numbers on a number line.
   b) To convert fractions to terminating or repeating decimals, and vice versa.
   c) To add and subtract rational numbers.
   d) To multiply and divide rational numbers.
   e) To use the order of operations in evaluating rational arithmetic expressions.

2. Exponents
   a) To evaluate a positive power of a numerical base.
   b) To evaluate multiplication and division of positive powers of the same numerical base.
   c) To perform the product and quotient exponent properties with variable bases.
   d) To write numbers in scientific notation and vice versa.
   e) To perform multiplication and division of numbers expressed in scientific notation.

3. Polynomials
   a) To add and subtract polynomials.
   b) To multiply a monomial by a monomial.
   c) To multiply a polynomial by a monomial.
   d) To multiply a binomial by a binomial.
   e) To divide a monomial by a monomial
   f) To divide a polynomial by a monomial.

Mathematics 20

Major Concepts	Number of Hours
Mathematics 20	8
A. Irrational Numbers ! Square root operations	
B. Consumer Mathematics ! Credit and Loans ! Income and Property Taxes	8
C. Polynomials and Rational Expressions ! Factoring - up to trinomials ($ax^2 + bx + c$) ! Laws of exponents - integral exponents ! Operations - (+, -, x, /)	15
D. Quadratic Functions ! Graphing various types of quadratic functions ! Determine properties of each constant for the types used in section a) ! Analyze and solve real-world problems based on a) and b)	10
E. Quadratic Equations ! Solve by factoring ! Solve by `square root property' ! Solve radical equations - one radicand ! Solve real-world problems based on a), b), c)	10
F. Probability ! Experimental ! Theoretical	4
G. Angles and Polygons ! Congruent triangles - informal, guided proofs ! Similar polygons ! Real-world problems ! Areas and volumes of similar figures	15-20
H. Circles ! Some relationships of tangents, chords, arcs ! Real-world problems	5
I. Optional Topics ! Consumer Price Index ! Insurance	
Total	75-80

! The remaining available time may be spent on additional practice, enrichment, or extension of the course.

Objectives

A. Irrational Numbers

Foundational Objective

! To identify an irrational number and to demonstrate the ability to add, subtract, multiply, and divide square root radicals. (10 01 01). Supported by the following learning objectives.

1. To define and illustrate, by means of examples, the term absolute value.

2. To express square root radicals as mixed radicals in simplest form.

3. To add, subtract, multiply, and divide square root radicals.

4. To rationalize monomial denominators.

B. Consumer Mathematics

Foundational Objectives

! To demonstrate an understanding of credit and to employ the appropriate mathematics in determining the cost to the consumer of various types of credit. (10 02 01). Supported by learning objectives 1 to 10.

! To display an awareness of the kinds of taxes encountered by the consumer, and to demonstrate the ability to calculate these taxes using the appropriate mathematics. (10 02 02). Supported by learning objectives 11 to 14.

1. To define credit and determine its appropriate use.

2. To compare the advantages and disadvantages of various credit cards.

3. To calculate the monthly interest charges and service charges on an unpaid credit card balance.

4. To identify and compare an instalment charge account and a thirty-day account.

5. To identify the characteristics of a personal loan.

6. To compare the cost of a consumer loan from various institutions.

7. To calculate the monthly payments for a loan, using formulas, tables, calculators, and computers.

8. a) To determine the total cost of a personal loan.
   b) To determine the percent of the total amount repaid (or borrowed) which is devoted to interest.

9. To determine the importance of a credit rating.

10. To describe how to establish a good credit rating.

11. To calculate mill rates and property taxes.

12. To calculate discounts or penalties on taxes due, depending on when they are paid.

13. To determine and calculate permissible deductions from total income.

14. To calculate the tax payable on taxable income.

C. Polynomials and Rational Expressions

Foundational Objectives

! To demonstrate the ability to factor polynomial expressions, including trinomials of the type $ax^2 + bx + c$. (10 03 01). Supported by learning objectives 1 and 6.

! To demonstrate the ability to correctly simplify expressions that contain positive and negative integral exponents. (10 03 02). Supported by learning objectives 3 and 4.

- To demonstrate the ability to add, subtract, multiply, and divide rational expressions with monomial denominators. (10 03 03). Supported by learning objectives 2, 5, 7, and 8.

1. To factor polynomials of the following types: common factor, grouping, difference of squares, trinomial squares, trinomials where a=1 and $a \neq 1$, and combinations of all preceding types.

2. To divide a polynomial by a binomial, by factoring, and by long division.

3. To evaluate powers with positive and negative exponents.

4. To simplify variable expressions with integral exponents using the following properties of exponents: product, quotient, power of a product, power of a quotient, negative exponent, and zero exponent.

5. To determine the non-permissible values for the variable in rational expressions.

6. To simplify rational expressions by factoring.

7. To multiply and divide rational expressions.

8. To add and subtract rational expressions involving like and unlike **monomial** denominators.

D. Quadratic Functions

Foundational Objectives

- To be aware that the graph of an equation in two variables, where only one variable is of degree two, is a parabola. (10 04 01). Supported by learning objective 1.

- To draw graphs of equations representing parabolas. (10 04 02). Supported by learning objectives 2 and 3.

- To demonstrate an ability to interpret an equation of the form $y=a(x-p)^2=q$, as to the effect the values of a, p, and q have on the graph. (10 04 03). Supported by learning objectives 4 and 5.

1. To define a quadratic function.

2. To identify, graph, and determine the properties of quadratic functions of the following forms: $f(x)=ax^2$, $f(x)=x^2+q$, $f(x)=(x-p)^2$, $f(x)=a(x-p)^2+q$.

3. To determine the domain and range from the graph of a quadratic function.

4. To analyze the graphs of quadratic functions that depict real-world situations.

5. To solve problems involving the graphs of quadratic functions that depict real-world situations.

E. Quadratic Equations

Foundational Objectives

- To demonstrate the ability to solve quadratic equations by factoring, and by taking the square root of both sides of an equation. (10 05 01). Supported by learning objectives 1, 2, and 3.

- To demonstrate the ability to solve equations containing one radical. (10 05 02). Supported by learning objectives 4 and 5.

1. To solve quadratic equations by a) factoring, and b) by taking the square roots of both sides of an equation.

2. To calculate the exact value of the length of a side of a right triangle using the Pythagorean Theorem.

3. To solve word problems involving quadratic equations.

4. To solve and verify radical equations containing one radicand.

5. To solve problems that involve equations which contain radicals.

F. Probability

Foundational Objective

! To appreciate the role of probability in understanding everyday situations. (10 06 01). Supported by the following learning objectives.

1. To list the sample space and events for a random experiment.

2. To calculate the experimental probability of simple events by performing repeated experiments.

3. To calculate the theoretical probability of an event, and the probability of its complement.

G. Angles and Polygons

Foundational Objectives

! To develop the ability to identify pairs of congruent triangles and to employ the congruence postulates SSS, SAS, ASA, AAS, or HL in guided proofs showing such congruences. (10 07 01). Supported by learning objectives 1 to 5.

! To demonstrate the ability to apply the concepts of similar polygons and scale factors to determine the surface area and/or volume of similar polygons or solids. (10 07 02). Supported by learning objectives 8 to 15.

! To provide a reasonable explanation for congruences of pairs of triangles, or for corresponding parts of congruent triangles. (10 07 03). Supported by learning objectives 6 and 7.

1. To informally and formally construct congruent angles, and congruent triangles.

2. To determine the properties of congruent triangles.

3. To identify and state corresponding parts of congruent triangles.

4. To determine whether triangles are congruent by SSS, SAS, ASA, AAS, or HL.

5. To prove that two triangles are congruent by supplying the statements and reasons in a guided deductive proof.

6. To prove triangles congruent by SSS, SAS, AAS, ASA, or HL in a two-column deductive proof or paragraph form.

7. To prove corresponding parts of congruent triangles are congruent.

8. To identify similar polygons.

9. To determine the measure of corresponding angles in two similar polygons.

10. To calculate the scale factor of two similar polygons.

11. To calculate the length of a missing side of two similar polygons.

12. To show that two triangles are similar by the Angle Angle Similarity Theorem (Postulate in some resource texts).

13. To calculate the length of a missing side in two similar right triangles.

14. To solve problems involving similar triangles, and other polygons.

15. To determine surface area and volumes of similar polygons or solids.

H. Circles

Foundational Objective

! To develop and display an understanding of certain relationships of the chords, tangents, and arcs of a circle. (10 08 01). Supported by the following learning objectives.

1. To define the measure of a minor arc, and to calculate the measure of a central angle.

2. To determine the relationship that exists between the following:

   - the radius of a circle and a tangent line drawn to it at the point of tangency;
   - two tangents drawn to a circle from the same point;
   - chords and arcs in the same circle or in congruent circles;
   - a diameter and a chord bisected by the diameter; and,
   - two chords that intersect inside a circle.

3. To solve problems based on the relationships stated in G. 2.

Mathematics A 30

Major Concepts	**Number of Hours**
Mathematics A 30	
A. Permutations and Combinations ! Fundamental counting principles ! Permutations and combinations ! Word problems based on the above	10
B. Data Analysis ! Data collection - simulations ! Box and whisker plots ! Problem solving	5
C. Polynomials and Rational Expressions ! Factoring sum and difference of cubes ! Factor and remainder theorems ! Operations with rational expressions ! Solving equations which include rational expressions	10
D. Exponents and Radicals ! Rational exponents ! Operations with radicals ! Solving quadratic equations ! Solving radical equations ! Problem solving	14
E. Relations and Functions ! Linear functions ! Slope ! Linear equations ! Quadratic functions ! Inverse variation	24
F. Systems of Linear Equations ! Solving systems of equations ! Identifying the type of system ! Solving associated word problems	7
G. Angles and Polygons ! Trigonometry	10
Total	80

! The remaining time (20 hours) may be spent on additional practice, enrichment, or extension of the course.

Objectives

A. Permutations and Combinations

Foundational Objectives

- ! To demonstrate the ability to determine the number of permutations in a given situation. (10 01 01). Supported by learning objectives 1 to 4.

- ! To demonstrate the ability to determine the number of combinations in a given situation. (10 01 02). Supported by learning objectives 5 and 6.

1. To apply the fundamental counting principles to determine the number of possibilities that exist in a given situation.

2. To determine the number of permutations of n objects, ($nPn=n!$)

3. To determine the number of permutations of n different objects, taken r at a time. (nPr)

4. To determine the number of permutations of n objects, not all different.

5. To determine the number of combinations of n objects, taken r at a time.

6. To determine the number of combinations formed from more than one subset.

B. Data Analysis

Foundational Objectives

- ! To demonstrate developed skills and understanding in collecting and displaying a set of data for a given situation. (10 02 01). Supported by learning objectives 1 to 4.

- ! To provide reasonable explanations of the interpretation of a set of data. (10 02 02). Supported by learning objectives 5 and 6.

1. To list, and describe, the methods used to collect data.

2. To obtain data for real-world situations by using simulations (such as Monte Carlo simulations).

3. To review the methods for determining the measures of central tendency.

4. To construct box and whisker plots from simulated data.

5. To define and utilize the concept of percentiles (including the first, second, and third quartiles).

6. To solve related problems using statistical inference.

C. Polynomials and Rational Expressions

Foundational Objectives

- ! To demonstrate ability in the addition, subtraction, multiplication, and division of rational expressions. (10 03 01). Supported by learning objectives 1 to 7.

- ! To demonstrate ability in solving equations involving rational expressions. (10 03 02). Supported by learning objectives 8 and 9.

1. To factor the difference of squares of special polynomials.

2. To factor the sum and difference of cubes.

3. To factor polynomials using the factor theorem.

4. To use the remainder theorem to determine the remainder when a polynomial is divided by ($x-r$).

5. To simplify rational expressions involving opposites.

6. To add and subtract rational expressions with polynomial denominators.

7. To multiply and divide rational expressions involving opposites.

8. To solve and verify linear equations in one variable involving rational algebraic expressions (including polynomial denominators).

9. To solve and verify the solutions of quadratic equations involving rational algebraic expressions.

9. To solve and verify equations involving absolute value.

10. To solve radical equations with two unlike radicands.

11. To solve word problems involving radical equations.

D. Exponents and Radicals

Foundational Objectives

! To be able to illustrate the relationship between the radical and exponential forms of an equation. (10 04 01). Supported by learning objectives 1, 3, and 5.

! To demonstrate the ability to work with operations involving radical numbers. (10 04 02). Supported by learning objectives 2, 4, 6, and 7.

! To be able to solve equations involving radicals, and to be able to justify the solutions. (10 04 03). Supported by learning objectives 8 to 11.

1. To evaluate powers with rational exponents.

2. To apply the laws of exponents to simplify expressions involving rational exponents.

3. To write exponential expressions in radical form.

4. To simplify square root and cube root expressions.

5. To write radical expressions in exponential form.

6. To add, subtract, multiply, and divide square root and cube root expressions.

7. To rationalize monomial and binomial denominators in radical expressions.

8. To solve and verify the solutions of quadratic equations by factoring, completing the trinomial square, and using the quadratic formula.

E. Relations and Functions

Foundational Objectives

! To be able to produce graphs of relations and functions, and to be able to denote which graphs represent functions. (10 05 01). Supported by learning objectives 1, 2, 3, 6, 10, 11, and 15.

! To demonstrate the ability to interpret graphs representing functions, and to identify key points of these graphs. (10 05 02). Supported by learning objectives 4, 5, 7, 8, 9, 12, 13, and 14.

! To be able to identify inverse variations, and to demonstrate solutions to problems involving inverse variations. (10 05 03). Supported by learning objectives 16 to 19.

1. To display a relation as a set of ordered pairs, mapping diagram, and a graph.

2. To graph relations representing real-world situations.

3. To determine the domain and range of a relation.

4. To determine if a set of ordered pairs represents a function.

5. To determine if a relation is a function by applying the vertical line test.

6. To determine if a function is one-to-one or many-to-one.

7. To calculate the distance between two ordered pairs in the coordinate plane.

8. To determine the coordinates of the midpoint of a segment.

9. To solve word problems involving medians, altitudes, and perpendicular bisectors.

10. To graph linear equations using the x- and y- intercepts.

11. To graph linear equations using one intercept and the slope.

12. To write the equation of a line in standard form using: two intercepts, slope and one intercept, one point and the equation of a parallel line, and one point and the equation of a perpendicular line.

13. To solve real-world problems involving quadratic functions by analyzing their graphs.

14. To graph quadratic functions of the standard form $f(x)=a(x-p)^2+q$, by determining the vertex, axis of symmetry, concavity, maximum or minimum values, domain range, and zeroes.

15. To graph quadratic functions of the general form $f(x)=ax^2+bx+c$, by completing the trinomial square and converting to one of the standard forms.

16. To identify and graph examples of inverse variation taken from real-world situations.

17. To state the domain and range, along with any restrictions, for the graphs of inverse variations.

18. To determine the constant of proportionality of an inverse relation.

19. To solve problems that involve inverse variation.

F. Systems of Linear Equations

Foundational Objectives

! To be able to identify the number of possible solutions of a system of linear equations. (10 06 01). Supported by learning objective 3.

! To demonstrate the ability to solve a system of linear equations. (10 06 02). Supported by learning objectives 1, 2, and 4.

1. To solve and verify systems of linear equations in two unknowns by the following methods: graphic, substitution, elimination, and augmented matrices.

2. To solve linear systems in two unknowns that have rational coefficients and verify the solutions.

3. To recognize the characteristics of linear equations in two variables with graphs that are inconsistent, consistent-dependent, or consistent-independent.

4. To solve word problems involving linear systems in two variables.

G. Angles and Polygons

Foundational Objectives

! To demonstrate the ability to determine trigonometric ratios in a given situation, and apply these ratios to solving real-world problems. (10 07 01). Supported by the following learning objectives.

1. To sketch an angle in standard position.

2. To determine the distance from the origin to a point on the terminal arm of an angle in standard position.

3. To determine the value of the six trigonometric ratios when given a point on the terminal arm of an angle in standard position (x, y, r).

4. To determine coterminal angles for a given angle.

5. To determine the reference angle for positive or negative angles.

6. To determine the values for the six trigonometric ratios, when given one trigonometric ratio and the quadrant in which the angle terminates.

7. To determine the values for the trigonometric ratios by using a calculator.

8. To apply the trigonometric ratios to problems involving right triangles.

9. To determine the relationships among the sides of each special right triangle (45-45-90 and 30-60-90).

10. To calculate the length of the missing sides of the special right triangles when given the exact value of one side.

Mathematics B 30

Major Concepts		Number of Hours
Mathematics B 30		
A. Probability ! Independent, dependent, mutually exclusive events ! Binomial Theorem		12
B. Data Analysis ! Data distributions - normal, skewed ! Standard deviation - calculation, interpretation ! z-scores - calculation and usage ! Problem solving		10
C. Matrices ! Operations with matrices ! Row operations ! Solving equations ! Linear Programming - Systems of Inequalities		15
D. Complex Numbers ! Operations with complex numbers		5
E. Quadratic Equations ! Quadratic Formula ! Nature of Roots ! Equations of degree greater than two ! Quadratic inequalities		10
F. Polynomial and Rational Functions ! Sketch and analyze ! Inverse of a function ! Reciprocal of a function		10
G. Exponential and Logarithmic Functions ! Laws of exponents ! Graphs of these functions ! Solving equations and problems ! Geometric sequences and series		18
! Problems	Total	80

! The remaining time (20 hours) may be spent on additional practice, enrichment, or extension of the course.

Objectives

A. Probability

Foundational Objectives

- ! To demonstrate the ability to set up and calculate probabilities of related events. (10 01 01). Supported by learning objectives 1 to 6.

- ! To apply the Binomial Theorem to expand binomials, and to real-world problems (10 01 02). Supported by learning objectives 7 and 9.

1. To define the principle of inclusion and exclusion when working with two or more sets and/or events.

2. To determine the number of permutations of n objects arranged in a circle.

3. To determine the probability of mutually exclusive events.

4. To determine the probability of two or more independent events.

5. To determine the probability of dependent events (conditional probabilities).

6. To set up, analyze, estimate, and solve word problems based on objectives 1-5.

7. To determine the coefficients of terms in a binomial expansion using the Binomial Theorem. (Pascal's Triangle or combinations could be used to introduce this topic.)

8. To expand binomials of the form $(a+b)^n$, using the Binomial Theorem.

9. To solve word problems associated with objectives 7 and 8.

B. Data Analysis

Foundational Objectives

- ! To determine the standard deviation of a set of data, and to utilize the standard deviation in analyzing that set of data. (10 02 01). Supported by learning objectives 1 to 3.

- ! To develop skill in interpreting data through the use of z-scores. (10 02 02). Supported by learning objectives 4 to 6.

1. To describe and illustrate normal and skewed distributions using real-world examples.

2. To calculate the standard deviation of a set of data.

3. To utilize the standard deviation to interpret data represented by a normal distribution.

4. To define and calculate z-scores.

5. To be able to utilize z-scores as an aid in interpreting data.

6. To solve related real-world problems using statistical inference.

C. Matrices

Foundational Objectives

- ! To illustrate appropriate real-world situations using matrices. (10 03 01). Supported by learning objective 2.

- ! To demonstrate knowledge of terms associated with matrices. (10 03 02). Supported by learning objectives 1 and 6.

- ! To develop skills in matrix operations and in solving related real-world problems. (10 03 03). Supported by learning objectives 3, 4, 5, 7, 8, 9, 10, 11, and 12.

1. To define basic terms associated with matrices.

2. To create a matrix to illustrate a given situation.

3. To add and subtract matrices.

4. To add and subtract matrices using scalar multiplication.

5. To multiply two matrices (not larger than 3x3).

6. To determine the properties of matrices with respect to addition, scalar multiplication and multiplication.

7. To use row operations with matrices.

8. To determine the inverse of a "2x2" matrix.

9. To solve matrix equations using multiplication by an inverse. (Orders higher than 2x2 could be solved using technology.)

10. To graph systems of inequalities.

11. To determine the points of intersection of lines drawn in objective 10.

12. To determine which vertices of the polygon formed by a system of inequalities maximizes or minimizes a given linear function.

D. Complex Numbers

Foundational Objective

! To demonstrate the skills developed in operations with complex numbers. (10 04 01). Supported by the following learning objectives.

1. To define and illustrate complex numbers.

2. To express complex numbers in the form a+bi.

3. To add and subtract complex numbers.

4. To multiply and divide complex numbers.

5. To divide complex numbers using conjugates.

E. Quadratic Equations

Foundational Objectives

! To demonstrate skill in solving quadratic equations. (10 05 01). Supported by learning objectives 1, 2, 3, 7, and 8.

! To write a quadratic equation through analysis of the given roots. (10 05 02). Supported by learning objectives 4, 5, and 6.

1. To solve quadratic equations using the quadratic formula.

2. To solve quadratic equations having complex roots.

3. To solve word problems involving real-world applications of quadratic equations.

4. To determine the nature of the roots of a quadratic equation, using the discriminant.

5. To determine that the sum of the roots of a quadratic equation $ax^2+bx+c = 0$ equals (-b/a), and that the product of the roots equals (c/a).

6. To write a quadratic equation, given the roots.

7. To solve equations of degree greater than two by expressing them in quadratic form. eg. $x^4-34x^2+225=0$.

8. To solve quadratic inequalities.

F. Polynomial and Rational Functions

Foundational Objectives

! To demonstrate the ability to graph, and to analyze the graphs, of polynomial and rational functions. (10 06 01). Supported by learning objectives 1 to 3.

! To demonstrate understanding of an inverse of a function. (10 06 02). Supported by learning objectives 4 and 5.

1. To define and illustrate polynomial and rational functions.

2. To sketch the graphs of polynomial and rational functions with integral coefficients, using calculators or computers.

3. To analyze the characteristics of the graphs of polynomial functions, and to identify the 'zeroes' of these graphs.

4. To define, determine, and sketch the inverse of a function, where it exists.

5. To define, determine, and sketch the reciprocal of a function.

G. Exponential and Logarithmic Functions

Foundational Objectives

! To develop skills and knowledge in working with a variety of exponential and logarithmic functions. (10 07 01). Supported by learning objectives 1 to 5.

! To demonstrate the ability to apply the knowledge of exponential and logarithmic functions to real-world situations. (10 07 02). Supported by learning objectives 6 to 17.

1. To define exponential functions and logarithmic functions.

2. To use the laws of exponents for integral and rational exponents.

3. To work with logs of numbers with bases other than 10.

4. To construct graphs of exponential functions and logarithmic functions, to identify the properties of these graphs, and to recognize they are inverses of each other.

5. To sketch graphs of exponential and logarithmic functions by selecting an appropriate point for the new origin.

6. To solve exponential and logarithmic equations.

7. To solve word problems involving exponential and logarithmic functions.

8. To identify a geometric sequence.

9. To determine the n^{th} term of a geometric sequence.

10. To calculate the required number of geometric means between given terms.

11. To calculate the sum of a geometric series.

12. To define and illustrate the following terms: geometric sequence, compound interest, present value, annuity, geometric means.

13. To determine the limit of a sequence.

14. To calculate the sum of an infinite series.

15. To solve word problems containing arithmetic or geometric series.

16. To solve word problems involving compound interest or present value.

17. To solve word problems involving annuities or mortgages.

Mathematics C 30

Major Concepts	Number of Hours
Mathematics C 30	
A. Mathematical Proof	23
! Deductive proof	
! Indirect proof	
! Induction	
B. Conic Sections	15
! Locus	
! Circle	
! Parabola	
! Ellipse	
! Hyperbola	
! Systems of Equations	
C. Circular Functions	15
! Radian measure	
! Arc length	
! Graphs of trigonometric functions	
D. Applications of Trigonometry	12
! Laws of Sines and Cosines	
! Solving triangles, including the ambiguous case	
! Problems	
! Areas of triangles	
E. Trigonometric Identities	10
! Basic identities	
! Sum and difference identities	
! Double, half-angle, and n.⊖ identities	
F. Trigonometric Equations	5
! Particular and general solutions	
Total	80

! The remaining time (20 hours) may be spent on additional practice, enrichment, or extension of the course.

Objectives

A. Mathematical Proof

Foundational Objective

! To appreciate the various types of mathematical thinking processes, and to demonstrate skill in applying these processes. (10 01 01). Supported by the following learning objectives.

1. To define and illustrate by means of examples: deductive, inductive, and analogical statements or arguments.

2. To complete deductive proofs from geometry, using a two-column format.

3. To complete proofs using some of the methods of co-ordinate geometry.

4. To redefine properties of numbers.

5. To prove trigonometric identities using a two-column format.

6. To prove solutions to algebraic exercises using a two-column format.

7. To introduce indirect proof and use it in several proofs.

8. To introduce the principle of mathematical induction.

9. To prove assertions by using mathematical induction.

B. Conic Sections

Foundational Objectives

! To become aware of the various conic sections, and to demonstrate skill in graphing and writing equations of the conic sections. (10 02 01). Supported by learning objectives 1 to 7.

! To demonstrate the ability to solve systems of linear-quadratic and quadratic-quadratic equations. (10 02 02). Supported by learning objectives 8 and 9.

1. Locus
   a) To define and illustrate a locus in a number of situations.
   b) To sketch and identify a locus given its description.

2. Circle
   a) To determine the equation of a circle in the general form $Ax^2+By^2+Cx+Dy+E=0$, when given the centre and radius.
   b) To change a given equation of a circle in the general form to the centre-radius form $r^2=(x-h)^2+(y-k)^2$, and to sketch and analyze its graph.
   c) To determine the equation of a circle in either of the forms $x^2+y^2=r^2$, or $(x-h)^2+(y-k)^2=r^2$ from the given data: centre and intercepts, end points of diameter, centre and a point on the circumference, centre and the equation of a tangent line, etc.

3. Parabola
   a) To determine the equation of a parabola in the general form $y-k=a(x-h)^2$ or $x-h=a(y-k)^2$ given the focus and directrix.
   b) To find the equation of a parabola with its vertex at the point (h, k) by replacing x by $x^1=x-h$ and replacing y by $y^1=y-k$ in the equation $x^2=4py$.
   c) To determine the equation of a parabola from the given data: focus and directrix, vertex and directrix, focus and vertex.
   d) To solve word problems involving the parabola.

4. Ellipse
   a) To define and illustrate the following terms: ellipse, foci, focal radii, major axis, minor axis, vertices, axis of symmetry.
   b) To determine the equation of an ellipse in the intercept form when given the foci and the sum of the focal radii. $x^2/a^2+y^2/b^2=1$
   c) To convert a given general equation for an ellipse to the intercept form and sketch and analyze its graph.

5. Hyperbola
   a) To define and illustrate the following terms: hyperbola, foci, focal radii, major axis, minor axis, axis of symmetry, asymptotes, vertices, rectangular hyperbola.
   b) To determine the equation of a hyperbola in the intercept form when given the foci and the difference of the focal radii.
   c) To convert a given general equation for a hyperbola to the intercept form, and sketch and analyze its graph.

6. a) To determine the equation of a hyperbola or an ellipse given sufficient information.
   b) To solve word problems involving ellipses or hyperbolas.

7. To examine the coefficients of the second degree equation $Ax^2+By^2+Cx+Dy+E=0$ and identify the conic section it represents.

8. To sketch diagrams to show possible relationships and intersections of the following systems: Linear-Quadratic; and Quadratic-Quadratic.

9. To solve the following systems of equations algebraically. Linear-Quadratic and Quadratic-Quadratic.

C. Circular Functions

Foundational Objectives

! To demonstrate the understanding of trigonometric functions as developed by circular functions. (10 03 01). Supported by learning objectives 1 to 5.

! To be able to produce the graphs of trigonometric functions. (10 03 02). Supported by learning objectives 6 and 7.

1. To define the trigonometric functions and real numbers by wrapping a number line around a circle.

2. To determine values of the primary and reciprocal trigonometric ratios.

3. To determine the radian measures of angles, to convert from radians to degrees, and vice versa.

4. To determine angular velocity and apply this concept in solving problems involving rotation.

5. To determine arc length and apply this in associated word problems.

6. To define and illustrate the following terms: periodic function, amplitude, domain, range, minimum value, maximum value, translation, wave motion, sinusoidal functions.

7. To state the range, period, amplitude, phase shift, minimum and maximum values, and sketch the graphs of:

   a) $y - k = a \sin b(x-h)$
   b) $y - k = a \cos b(x-h)$
   c) $y - k = a \tan b(x-h)$

D. Applications of Trigonometry

Foundational Objectives

! To demonstrate the ability to apply trigonometry to real-world problem situations. (10 04 01). Supported by learning objectives 1 to 5.

! To demonstrate the ability to calculate areas of given triangles using trigonometry. (10 04 02). Supported by learning objectives 6 and 7.

1. To define and illustrate the following terms: angles of elevation and depression, heading, bearing, compass direction.

2. To solve right triangles and associated word problems.

3. To solve oblique triangles by the use of the Law of Sines/Cosines.

4. To solve triangles including all solutions given two sides and a non-included angle (the Ambiguous Case).

5. To solve word problems by means of the Law of Sines/Cosines.

6. To determine the area of a triangle using K =½ ab sin C, K = a^2 sin B sin C / 2 sin A, or Heron's Formula -

$$K = \sqrt{s(s-a)(s-b)(s-c)} \; .$$

7. To solve word problems involving Objective D6.

E. Trigonometric Identities

Foundational Objective

! To demonstrate the ability to work with trigonometric identities and to be able to apply them when necessary. (10 05 01). Supported by the following learning objectives.

1. To prove and apply the reciprocal identities.

2. To prove and apply the quotient identities.

3. to prove and apply the Pythagorean identities.

4. To prove and apply the Addition/Subtraction identities.

5. To prove and apply the Double-Angle identities.

6. To determine sin n0, where n is a natural number

7. To apply the Half-Angle identities.

F. Trigonometric Equations

Foundational Objectives

! To demonstrate understanding and ability in solving trigonometric equations. (10 06 01). Supported by the following learning objective.

1. To solve trigonometric equations by finding a particular solution and by finding the general solution.

Flowchart for Secondary Mathematics (Core): Strands and Concepts

Secondary Strands	Data/ Consumer	Number Operations	Equations Problems	Algebra	Functions	Geometry	Trigonometry
Math C 30		Mathematical Proof	Trigonometry Equations	Trigonometry Identities	Circular Functions	Conic Sections	Trigonometry Applications
Math B 30	Data Analysis	Complex Numbers Probability	Quadratic Equations	Matrices Application of Functions	Exp., Log Functions Poly., Rat. Functions		
Math A 30	Data Analysis	Exponents, Radicals Permutations Combinations	Quadratic Equations Systems Linear Eq.	Polynomials Rational Expressions	Relations Functions	Angles, Polygons	6 Trig* Ratios (x,y,r)
Math 20	Consumer ! Credit ! Taxes	Irrational Numbers Probability	Quadratic Equations	Polynomials Rational Expressions	Quadratic Functions	Angles, Polygons Circles	Similarity* Scale Factor
Math 10	Consumer ! Income ! Budget		Equations Inequalities 1 Variable Linear	Algebra Numbers Polynomials	Relations, Functions	Angles, Polygons Lines, Line Segs.	Pyth. Th.* Primary Trig. Ratios
Middle Level Strands	Data Management	Numbers Operations	Problem Solving	Algebra	Ratio Proportion	Geometry/Measurement	

*Not a separate unit

Scope and Sequence for Secondary Mathematics (by strand)

The code in the column under the course of study refers to the Concept (capital letter) and the specific learning objective (number) under the Concept in that course of study.

For example, if the code C.6 appears in the 10 column under the course of study, this would indicate that it is in the Math 10 curriculum guide, the sixth learning objective in Concept C.

A ✓ in the Grade 9 column indicates this topic has been introduced in the middle level program.

Strand: Data Analysis/Consumer Mathematics

Learning Objectives	Course of Study					
	9	10	20	A 30	B 30	C 30
1. To calculate weekly gross wages involving regular pay, overtime pay, and piecework earnings.		C.1				
2. To calculate earnings for straight commission, or base wage plus commission.		C.2				
3. To determine the difference between gross pay and net pay.		C.3				
4. To calculate weekly, monthly, and yearly net pay.		C.4				
5. To define and explain the purpose of a budget.		C.5				
6. To determine and calculate monthly fixed expenditures.		C.6				
7. To investigate the guidelines in developing a budget.		C.7				
8. To plan a budget based on percentages allotted to various categories as suggested by financial institutions.		C.8				
9. To calculate the portion of total income spent on each category using percents.	✓	C.9				
10. To draw graphs (including circle graphs) representing budget figures, using appropriate software.	✓	C.10				
11. To calculate the actual amount of money to be spent on each category using the predetermined percentages.	✓	C.11				
12. To adjust a budget to changes in expenses.		C.12				
13. To define credit and determine its appropriate use.			B.1			

Strand: Data Analysis/Consumer Mathematics

Learning Objectives	Course of Study					
	9	10	20	A 30	B 30	C 30
14. To compare the advantages and disadvantages of various credit cards.			B.2			
15. To calculate the monthly interest charges and service charges on an unpaid credit card balance.			B.3			
16. To identify and compare an instalment charge account and a thirty-day account.			B.4			
17. To identify the characteristics of a personal loan.			B.5			
18. To compare the cost of a consumer loan from various institutions.	✓		B.6			
19. To calculate the monthly payments for a loan; using formulas, tables, calculators, and computers.	✓		B.7			
20. a) To determine the total cost of a personal loan. b) To determine the percent of the total amount repaid (or borrowed) which is devoted to interest.	✓		B.8			
21. To determine the importance of a credit rating.			B.9			
22. To describe how to establish a good credit rating.			B.10			
23. To calculate mill rates and property taxes.			B.11			
24. To calculate discounts or penalties on taxes due, depending on when they are paid.			B.12			
25. To determine and calculate permissible deductions from total income.			B.13			
26. To calculate the tax payable on taxable income.			B.14			

Strand: Data Analysis/Consumer Mathematics

Learning Objectives	Course of Study					
	9	10	20	A 30	B 30	C 30
27. To list, and describe, the methods used to collect data.	✓			B.1		
28. To obtain data for real-world situations by using simulations (such as Monte Carlo simulations).				B.2		
29. To review the methods for determining the measures of central tendency.	✓			B.3		
30. To construct box and whisker plots from simulated data.				B.4		
31. To define, and utilize, the concept of percentiles (including the first, second, and third quartiles).				B.5		
32. To solve related problems using statistical inference.				B.6		
33. To describe and illustrate normal and skewed distributions using real-world examples					B.1	
34. To calculate the standard deviation of a set of data.					B.2	
35. To utilize the standard deviation to interpret data represented by a normal distribution.					B.3	
36. To define and calculate z-scores.					B.4	
37. To be able to utilize z-scores as an aid in interpreting data.					B.5	
38. To solve related real-world problems using statistical inference.					B.6	

Strand: Numbers and Operations

Learning Objectives	Course of Study					
	9	10	20	A 30	B 30	C 30
1. To list the sample space and events for a random experiment.	✓		F.1			
2. To calculate the experimental probability of simple events by performing repeated experiments.	✓		F.2			
3. To calculate the theoretical probability of an event, and the probability of its complement.			F.3			
4. To define and illustrate, by means of examples, the term absolute value.			A.1			
5. To express square root radicals as mixed radicals in simplest form.			A.2			
6. To add, subtract, multiply, and divide square root radicals.			A.3			
7. To rationalize monomial denominators.			A.4			
8. To apply the fundamental counting principles to determine the number of possibilities that exist in a given situation.	✓			A.1		
9. To determine the number of permutations of n objects, (nPn=n!).				A.2		
10. To determine the number of permutations of n different objects, taken r at a time, (nPr).				A.3		
11. To determine the number of permutations of n objects, not all different.				A.4		
12. To determine the number of combinations of n objects, taken r at a time.				A.5		
13. To determine the number of combinations formed from more than one subset.				A.6		
14. To evaluate powers with rational exponents.				D.1		

Strand: Numbers and Operations

Learning Objectives	Course of Study					
	9	10	20	A 30	B 30	C 30
15. To apply the laws of exponents to simplify expressions involving rational exponents.				D.2		
16. To write exponential expressions in radical form.				D.3		
17. To simplify square root and cube root expressions.				D.4		
18. To write radical expressions in exponential form.				D.5		
19. To add, subtract, multiply, and divide square root and cube root expressions.				D.6		
20. To rationalize monomial and binomial denominators in radical expressions.				D.7		
21. To define the principle of inclusion and exclusion when working with two or more sets and/or events.					A.1	
22. To determine the number of permutations of n objects arranged in a circle.					A.2	
23. To determine the probability of mutually exclusive events.					A.3	
24. To determine the probability of two or more independent events.					A.4	
25. To determine the probability of dependent events (conditional probabilities).					A.5	
26. To set up, analyze, estimate, and solve word problems based on objectives 1-5.					A.6	
27. To determine the coefficients of terms in a binomial expansion using the Binomial Theorem. (Pascal's Triangle or combinations could be used to introduce this topic.)					A.7	
28. To expand binomials of the form $(a+b)^n$, using the Binomial Theorem.					A.8	

Strand: Numbers and Operations

Learning Objectives	Course of Study					
	9	10	20	A 30	B 30	C 30
29. To solve word problems associated with objectives 7 and 8.					A.9	
30. To define and illustrate complex numbers.					D.1	
31. To express complex numbers in the form a+bi.					D.2	
32. To add and subtract complex numbers.					D.3	
33. To multiply and divide complex numbers.					D.4	
34. To divide complex numbers using conjugates.					D.5	
35. To define and illustrate by means of examples: deductive, inductive, and analogical statements or arguments.						A.1
36. To complete deductive proofs from geometry, using a two-column format.						A.2
37. To complete proofs using some of the methods of co-ordinate geometry.						A.3
38. To re-define properties of numbers.						A.4
39. To prove trigonometric identities, using a two-column format.						A.5
40. To prove solutions to algebraic exercises, using a two-column format.						A.6
41. To introduce indirect proof and use it in several proofs.						A.7
42. To introduce the principle of mathematical induction.						A.8
43. To prove assertions by using mathematical induction.						A.9

Strand: Equations, Problems

Learning Objectives	Course of Study					
	9	10	20	A 30	B 30	C 30
1. To solve and verify the following types of equations in one variable by applying formal algebraic rules: equations containing variables on both sides of the equal sign, equations containing parentheses, equations containing fraction, or decimal coefficients.	✓	A.1				
2. To solve a formula for an indicated variable.	✓	A.2				
3. To solve, graph, and verify linear inequalities in one variable.		A.3				
4. To translate English phrases into mathematical terms and vice versa.	✓	A.4				
5. To solve real-world problems using various problem solving strategies.	✓	A.5				
6. To solve quadratic equations by: a) factoring, and b) by taking the square roots of both sides of an equation.			E.1			
7. To calculate the exact value of the length of a side of a right triangle using the Pythagorean Theorem.	✓		E.2			
8. To solve word problems involving quadratic equations.			E.3			
9. To solve and verify radical equations containing one radicand.			E.4			
10. To solve problems that involve equations which contain radicals.			E.5			
11. To solve and verify the solutions of quadratic equations by factoring, completing the trinomial square, and using the quadratic formula.				D.8		
12. To solve and verify equations involving absolute value.				D.9		
13. To solve radical equations with two unlike radicands.				D.10		

Strand: Equations, Problems

Learning Objectives	Course of Study					
	9	10	20	A 30	B 30	C 30
14. To solve word problems involving radical equations.				D.11		
15. To solve and verify systems of linear equations in two unknowns by the following methods: graphic, substitution, elimination, and augmented matrices.				F.1		
16. To solve linear systems in two unknowns that have rational coefficients and verify the solutions.				F.2		
17. To recognize the characteristics of linear equations in two variables with graphs that are inconsistent, consistent-dependent, or consistent-independent.				F.3		
18. To solve word problems involving linear systems in two variables.				F.4		
19. To solve quadratic equations using the quadratic formula.					E.1	
20. To solve quadratic equations having complex roots.					E.2	
21. To solve word problems involving real-world applications of quadratic equations.					E.3	
22. To determine the nature of the roots of a quadratic equation using the discriminant.					E.4	
23. To determine that the sum of the roots of a quadratic equation $ax^2+bx+c=0$ equals (-b/a), and that the product of the roots equals (c/a).					E.5	
24. To write a quadratic equation, given the roots.					E.6	

Strand: Equations, Problems

Learning Objectives	Course of Study					
	9	10	20	A 30	B 30	C 30
25. To solve equations of degree greater than two by expressing them in quadratic form. eg. $x^4-34x^2+225=0$.					E.7	
26. To solve quadratic inequalities.					E.8	
27. To solve trigonometric equations by finding a particular solution and by finding the general solution.						F.1

Strand: Algebra

Learning Objectives	Course of Study					
	9	10	20	A 30	B 30	C 30
1. Numbers and Operations		F.1				
a) To represent numbers on a number line.	✓					
b) To convert fractions to terminating or repeating decimals and vice versa.	✓					
c) To add and subtract rational numbers.	✓					
d) To multiply and divide rational numbers.	✓					
e) To use the order of operations in evaluating rational arithmetic expressions.	✓					
2. Exponents		F.2				
a) To evaluate a positive power of a numerical base.	✓					
b) To evaluate multiplication and division of positive powers of the same numerical base.	✓					
c) To perform the product and quotient exponent properties with variable bases.	✓					
d) To write numbers in scientific notation and vice versa.	✓					
e) To perform multiplication and division of numbers expressed in scientific notation.	✓					
3. Polynomials		F.3				
a) To add and subtract polynomials.	✓					
b) To multiply a monomial by a monomial.	✓					
c) To multiply a polynomial by a monomial.	✓					
d) To multiply a binomial by a binomial.						
e) To divide a monomial by a monomial.	✓					
f) To divide a polynomial by a monomial.						
4. To factor polynomials of the following types: common factor, grouping, difference of squares, trinomial squares, trinomials where $a=1$, trinomials where $a\neq1$, and combinations of all preceding types.			C.1			

Strand: Algebra

Learning Objectives	Course of Study					
	9	10	20	A 30	B 30	C 30
5. To divide a polynomial by a binomial by factoring and by long division.			C.2			
6. To evaluate powers with positive and negative exponents.			C.3			
7. To simplify variable expressions with integral exponents using the following properties of exponents: product, quotient, power of a product, power of a quotient, negative exponent, and zero exponent.			C.4			
8. To determine the non-permissible values for the variable in rational expressions.			C.5			
9. To simplify rational expressions by factoring.			C.6			
10. To multiply and divide rational expressions.			C.7			
11. To add and subtract rational expressions involving like and unlike **monomial** denominators.			C.8			
12. To factor the difference of squares of special polynomials.				C.1		
13. To factor the sum and difference of cubes.				C.2		
14. To factor polynomials using the factor theorem.				C.3		
15. To use the remainder theorem to determine the remainder when a polynomial is divided by (x-r).				C.4		
16. To simplify rational expressions involving opposites.				C.5		
17. To add and subtract rational expressions with polynomial denominators.				C.6		
18. To multiply and divide rational expressions involving opposites.				C.7		

Strand: Algebra

Learning Objectives	Course of Study					
	9	10	20	A 30	B 30	C 30
19. To solve and verify linear equations in one variable involving rational algebraic expressions (including polynomial denominators).				C.8		
20. To solve and verify the solutions of quadratic equations involving rational algebraic expressions.				C.9		
21. To define basic terms associated with matrices.					C.1	
22. To create a matrix to illustrate a given situation.					C.2	
23. To add and subtract matrices.					C.3	
24. To add and subtract matrices using scalar multiplication.					C.4	
25. To multiply two matrices (not larger than 3x3).					C.5	
26. To determine the properties of matrices with respect to addition, scalar multiplication, and multiplication.					C.6	
27. To use row operations with matrices.					C.7	
28. To determine the inverse of a `2x2' matrix.					C.8	
29. To solve matrix equations using multiplication by an inverse. (Orders higher than 2x2 could be solved using technology.)					C.9	
30. To graph systems of inequalities.					C.10	
31. To determine the points of intersection of lines drawing in objective 10.					C.11	
32. To determine which vertices of the polygon formed by a system of inequalities maximizes or minimizes a given linear function.					C.12	

Strand: Algebra

Learning Objectives	Course of Study					
	9	10	20	A 30	B 30	C 30
33. To identify, describe, and interpret examples of direct variation in real-world situations.	✓	B.19				
34. To solve proportions involving direct variation.	✓	B.20				
35. To solve problems involving direct variation.	✓	B.21				
36. To identify partial variation.		B.22				
37. To solve problems involving partial variation.		B.23				
38. To define, illustrate, and identify an arithmetic sequence.		B.24				
39. To determine the n^{th} term of an arithmetic sequence.		B.25				
40. To define arithmetic means, and to determine the required arithmetic means between given terms.		B.26				
41. To calculate the sum of an arithmetic series.		B.27				
42. To determine the constant of proportionality of an inverse relation.				E.18		
43. To solve problems that involve inverse variation.				E.19		
44. To identify a geometric sequence.					G.8	
45. To determine the n^{th} term of a geometric sequence.					G.9	
46. To calculate the required number of geometric means between given terms.					G.10	
47. To calculate the sum of a geometric series.					G.11	

Strand: Algebra

Learning Objectives	Course of Study					
	9	10	20	A 30	B 30	C 30
48. To define and illustrate the following terms: geometric sequence, compound interest, present value, annuity, geometric means.					G.12	
49. To determine the limit of a sequence.					G.13	
50. To calculate the sum of an infinite series.					G.14	
51. To solve word problems containing arithmetic or geometric series.					G.15	
52. To solve word problems involving compound interest or present value.					G.16	
53. To solve word problems involving annuities or mortgages.					G.17	
54. To prove and apply the reciprocal identities.						E.1
55. To prove and apply the quotient identities.						E.2
56. To prove and apply the Pythagorean identities.						E.3
57. To prove and apply the Addition/Subtraction identities.						E.4
58. To prove and apply the Double-Angle identities.						E.5
59. To determine sin n.θ, where n is a natural number.						E.6
60. To apply the Half-Angle identities.						E.7

Strand: Functions

Learning Objectives	Course of Study					
	9	10	20	A 30	B 30	C 30
1. To define the following terms: relation, ordered pair, abscissa, ordinate, function, linear function, slope, x-intercept, ratio, proportion, direct variation, partial variation.	✓	B.1				
2. To identify and express examples of relations in the real world.	✓	B.2				
3. To graph ordered pairs in the Cartesian coordinate plane.	✓	B.3				
4. To graph real-world relations in the Cartesian coordinate plane.	✓	B.4				
5. To read information from a graph.	✓	B.5				
6. To identify, graph and interpret examples of linear functions describing real-world situations.	✓	B.6				
7. To graph a linear function using a table of values.	✓	B.7				
8. To determine if a relation is a function by employing the vertical line test.		B.8				
9. To solve equations in two variables given the domain of one of the variables.		B.9				
10. To determine if an ordered pair is a solution to the linear equation.	✓	B.10				
11. To calculate the slope of a line graphically and algebraically when given two points that lie on the line.		B.11				
12. To determine the slope of horizontal lines, vertical lines, parallel lines, and perpendicular lines.		B.12				
13. To graph a linear equation using the x- and y-intercept.		B.13				

Strand: Functions

Learning Objectives	Course of Study					
	9	10	20	A 30	B 30	C 30
14. To graph a linear equation using the slope and y-intercept.		B.14				
15. To write the equation of a line when given the slope and one point on the line.		B.15				
16. To write the equation of a line when given two points that lie on the line.		B.16				
17. To construct scatterplots from real-world data.		B.17				
18. To interpret and critically analyze these constructed scatterplots.		B.18				
19. To define a quadratic function.			D.1			
20. To identify, graph, and determine the properties of quadratic functions of the following forms: $f(x)=ax^2$, $f(x)=x^2+q$, $f(x)=(x-p)^2$, $f(x)=a(x-p)^2+q$.			D.2			
21. To determine the domain and range from the graph of a quadratic function.			D.3			
22. To analyze the graphs of quadratic functions that depict real-world situations.			D.4			
23. To solve problems involving the graphs of quadratic functions that depict real-world situations.			D.5			
24. To display a relation as a set of ordered pairs, mapping diagram, and a graph.	✓			E.1		
25. To graph relations representing real-world situations.	✓			E.2		
26. To determine the domain and range of a relation.				E.3		
27. To determine if a set of ordered pairs represents a function.				E.4		

Strand: Functions

Learning Objectives	Course of Study					
	9	10	20	A 30	B 30	C 30
28. To determine if a relation is a function by applying the vertical line test.				E.5		
29. To determine if a function is one-to-one or many-to-one.				E.6		
30. To calculate the distance between two ordered pairs in the coordinate plane.				E.7		
31. To determine the coordinates of the midpoint of a segment.				E.8		
32. To solve word problems involving medians, altitudes, and perpendicular bisectors.				E.9		
33. To graph linear equations using the x- and y-intercepts.				E.10		
34. To graph linear equations using one intercept and the slope.				E.11		
35. To write the equation of a line in standard form using: two intercepts, slope and one intercept, one point and the equation of a parallel line, and one point and the equation of a perpendicular line.				E.12		
36. To solve real-world problems involving quadratic functions by analyzing their graphs.				E.13		
37. To graph quadratic functions of the standard form $f(x)=a(x-p)^2+q$, by determining the vertex, axis of symmetry, concavity, maximum or minimum values, domain, range, and zeroes.				E.14		
38. To graph quadratic functions of the general form $f(x)=ax^2+bx+c$, by completing the trinomial square and converting to one of the standard forms.				E.15		
39. To identify and graph examples of inverse variation taken from real-world situations.				E.16		

Strand: Functions

Learning Objectives	Course of Study					
	9	10	20	A 30	B 30	C 30
40. To state the domain and range, along with any restrictions, for the graphs of inverse variations.				E.17		
41. To define and illustrate polynomial and rational functions.					F.1	
42. To sketch the graphs of polynomial and rational functions with integral coefficients, using calculators or computers.					F.2	
43. To analyze the characteristics of the graphs of polynomial functions, and to identify the 'zeroes' of these graphs.					F.3	
44. To define, determine, and sketch the inverse of a function, where it exists.					F.4	
45. To define, determine, and sketch the reciprocal of a function.					F.5	
46. To define exponential functions and logarithmic functions.					G.1	
47. To use correctly the laws of exponents for integral and rational exponents.					G.2	
48. To work with logs of numbers with bases other than 10.					G.3	
49. To construct graphs of exponential functions and logarithmic functions, to identify the properties of these graphs, and to recognize they are inverses of each other.					G.4	
50. To sketch graphs of exponential and logarithmic functions by selecting an appropriate point for the new origin.					G.5	
51. To solve exponential and logarithmic equations.					G.6	
52. To solve word problems involving exponential and logarithmic functions.					G.7	

Strand: Functions

Learning Objectives	Course of Study					
	9	10	20	A 30	B 30	C 30
53. To define the trig functions of real numbers by wrapping a number line around a circle.						C.1
54. To determine values of the primary and reciprocal trigonometric ratios.						C.2
55. To determine the radian measures of angles, to convert from radians to degrees and vice versa.						C.3
56. To determine angular velocity and apply this concept in solving problems involving rotation.						C.4
57. To determine arc length and apply this in associated word problems.						C.5
58. To define and illustrate the following terms: period function, amplitude, domain, range, minimum value maximum value, translation, wave motion, sinusoidal functions.						C.6
59. To state the range, period, amplitude, phase shift, minimum and maximum values, and sketch the graphs of: a) y-k=a sin b(x-h) b) y-k=a cos b(x-h) c) y-k=a tan b(x-h)						C.7

Strand: Geometry

Learning Objectives	Course of Study					
	9	10	20	A 30	B 30	C 30
1. To define: line segment, ray, line, bisector, perpendicular line, perpendicular bisector, transversal, alternate interior angles, corresponding angles, same-side interior angles.	✓	D.1				
2. To identify and calculate the measures of the following angles formed by parallel lines, corresponding angles, alternate interior angles, and same-side interior angles.	✓	D.2				
3. To solve word problems involving angles formed by parallel lines.	✓	D.3				
4. To informally construct a line parallel to a given line through a point not on the line.	✓	D.4				
5. To informally construct a line perpendicular to a given line from a point not on the line.	✓	D.5				
6. To informally construct a line perpendicular to a given line through a point on the line.	✓	D.6				
7. To informally construct the perpendicular bisector of a line segment.	✓	D.7				
8. To define and illustrate by drawing the following: acute angle, right angle, obtuse angle, straight angle, reflex angle, complementary angles, supplementary angles, adjacent angles, vertically opposite angles, congruent angles, central angles of a regular polygon.	✓	E.1				
9. To solve word problems involving the angles stated in E.1.		E.2				
10. To define and illustrate the following polygons: convex, non-convex, regular, quadrilateral parallelogram, rectangle, rhombus, square, trapezoid, isosceles trapezoid.	✓	E.3				

Learning Objectives	Course of Study					
	9	10	20	A 30	B 30	C 30
11. To classify quadrilaterals as trapezoids, isosceles trapezoids, parallelograms, rectangles, rhombuses, and squares.	✓	E.4				
12. To informally construct parallelograms, rectangles, rhombuses, and squares.	✓	E.5				
13. To state and apply the properties of parallelograms.	✓	E.6				
14. To determine the sum of the measures of the interior and exterior angles of a convex polygon of n sides.		E.7				
15. To determine the measure of a central angle in a regular n-gon.		E.8				
16. To determine the measures of the interior and exterior angles of regular n-gons.		E.9				
17. To determine the number of diagonals in a polygon of n sides.		E.10				
18. To define the measure of a minor arc, and to calculate the measure of a central angle.			H.1			
19. To determine the relationship that exists between the following: • the radius of a circle and a tangent line drawn to it at the point of tangency; • two tangents drawn to a circle from the same point; • chords and arcs in the same circle or in congruent circles; • a diameter and a chord bisected by the diameter; and, • two chords that intersect inside a circle.			H.2			
20. To solve problems based on the relationships state in G.2.			H.3			
21. To informally and formally construct congruent angles, and congruent triangles.	✓		G.1			

Strand: Geometry

Learning Objectives	Course of Study					
	9	10	20	A 30	B 30	C 30
22. To determine the properties of congruent triangles.	✓		G.2			
23. To identify and state corresponding parts of congruent triangles.	✓		G.3			
24. To determine whether triangles are congruent by SSS, SAS, ASA, AAS, or HL.	✓		G.4			
25. To prove that two triangles are congruent by supplying the statements and reasons in a guided deductive proof.	✓		G.5			
26. To prove triangles congruent by SSS, SAS, AAS, ASA, or HL in a two-column deductive proof or paragraph form.			G.6			
27. To prove corresponding parts of congruent triangles are congruent.			G.7			
28. Locus a) To define and illustrate a locus in a number of situations. b) To sketch and identify a locus given its description.						B.1
29. Circle a) To determine the equation of a circle in the general form $Ax^2+By^2+Cx+Dy+E=0$, when given the centre and radius. b) To change a given equation of a circle in the general form to the centre-radius form $r^2=(x-h)^2+(y-k)^2$, and to sketch and analyze its graph. c) To determine the equation of a circle in either of the forms $x^2+y^2=r^2$, or ($r^2=(x-h)^2+(y-k)^2$ from the given data: centre and intercepts, end points of diameter, and centre and a point on the circumference, centre and the equation of a tangent line, etc.						B.2

Learning Objectives	Course of Study					
	9	10	20	A 30	B 30	C 30
30. Parabola a) To determine the equation of a parabola in the general form $y-k=a(x-h)^2$ or $x-h=a(y-k)^2$ given the focus and directrix. b) To find the equation of a parabola with its vertex at the point (h,k) by replacing x by $x^1=x-h$ and replacing y by $y^1 = y-k$ in the equation $x^2=4py$. c) To determine the equation of a parabola from the given data: focus and directrix, vertex and directrix, focus and vertex.						B.3
31. Ellipse a) To define and illustrate the following terms: ellipse, foci, focal radii, major axis, minor axis, vertices, axis of symmetry. b) To determine the equation of an ellipse in the intercept form when given the foci and the sum of the focal radii, $x^2/a^2+y^2/b^2=1$. c) To convert a given general equation for an ellipse to the intercept form and sketch and analyze its graph.						B.4
32. Hyperbola a) To define and illustrate the following terms: hyperbola, foci, focal radii, major axis, minor axis, axis of symmetry, asymptotes, vertices, rectangular hyperbola. b) To determine the equation of a hyperbola in the intercept form when given the foci and the difference of the focal radii. c) To convert a given general equation for a hyperbola to the intercept form, and sketch and analyze its graph.						B.5
33. a) To determine the equation of a hyperbola or an ellipse, given sufficient information. b) To solve word problems involving ellipses or hyperbolas.						B.6

Strand: Geometry

Learning Objectives	Course of Study					
	9	10	20	A 30	B 30	C 30
34. To examine the coefficients of the second degree equation $Ax^2+By^2+Cx+Dy+E=0$, and identify the conic section it represents.						B.7
35. To sketch diagrams to show possible relationships and intersections of the following systems: Linear-Quadratic; and Quadratic-Quadratic.						B.8
36. To solve the following systems of equations algebraically: Linear-Quadratic and Quadrati-Quadratic.						B.9

Strand: Trigonometry

Learning Objectives	Course of Study					
	9	10	20	A 30	B 30	C 30
1. To calculate to two decimal places the length of a missing side of a right triangle using the Pythagorean Theorem.	✓	E.11				
2. To solve word problems using the Pythagorean Theorem.	✓	E.12				
3. To determine if a triangle is a right triangle by using the converse of the Pythagorean Theorem.		E.13				
4. To determine the value of the three primary trigonometric ratios using a calculator.		E.14				
5. To determine the measure of an angle given the value of one trigonometric ratio of the angle by using a calculator.		E.15				
6. To calculate the measure of an angle or the length of a side of a right triangle using the tangent, sine and cosine ratios.		E.16				
7. To solve word problems that involve trigonometric ratios using a calculator.		E.17				
8. To identify similar polygons.	✓		G.8			
9. To determine the measure of corresponding angles in two similar polygons.	✓		G.9			
10. To calculate the scale factor of two similar polygons.	✓		G.10			
11. To calculate the length of a missing side of two similar polygons.			G.11			
12. To show that two triangles are similar by the Angle Angle Similarity Theorem (Postulate in some resource texts).			G.12			
13. To calculate the length of a missing side in two similar right triangles.			G.13			
14. To solve problems involving similar triangles and other polygons.			G.14			

Strand: Trigonometry

Learning Objectives	Course of Study					
	9	10	20	A 30	B 30	C 30
15. To determine surface area and volumes of similar polygons or solids.	✓		G.15			
16. To sketch an angle in standard position.				G.1		
17. To determine the distance from the origin to a point on the terminal arm of an angle in standard position.				G.2		
18. To determine the value of the six trigonometric ratios when given a point on the terminal arm of an angle in standard position (x,y,r).				G.3		
19. To determine coterminal angles for a given angle.				G.4		
20. To determine the reference angle for positive or negative angles.				G.5		
21. To determine the values of the six trigonometric ratios, when given one trigonometric ratio and the quadrant in which the angle terminates.				G.6		
22. To determine the values for the trigonometric ratios by using a calculator.				G.7		
23. To apply the trigonometric ratios to problems involving right triangles.				G.8		
24. To determine the relationships among the sides of each special right triangle (45-45-90 and 30-60-90).				G.9		
25. To calculate the length of the missing sides of the special right triangles when given the exact value of one side.				G.10		
26. To define and illustrate the following terms: angles of elevation and depression, heading, bearing, compass direction.						D.1
27. To solve right triangles and associated word problems.						D.2

Strand: Trigonometry

Learning Objectives	Course of Study					
	9	10	20	A 30	B 30	C 30
28. To solve oblique triangles by the use of the Law of Sines/Cosines.						D.3
29. To solve triangles including all solutions given two sides and a non-included angle (the Ambiguous Case).						D.4
30. To solve word problems by means of the Law of Sines/Cosines.						D.5
31. To determine the area of a triangle using $K=\frac{1}{2}ab\sin C$, $K=a^2\sin B\sin C/2\sin A$, or Heron's Formula - $K=\sqrt{s(s-a)(s-b)(s-c)}$.						D.6
32. To solve word problems involving Objective D6.						D.7

Templates for Assessment and Evaluation

The templates included represent some examples of the various types of evaluation and assessment the teacher might employ. The teacher is encouraged to peruse *Student Evaluation: A Teacher Handbook* (1991) and curriculum guides from other subject areas in order to obtain a more comprehensive set of evaluation templates. In addition, the teacher may adapt any of the templates to accommodate students in the classroom.

Mathematics 10

Algebraic Skills

Student	Numbers and Operation				Exponents				Polynomials															
	Add Rational Numbers	Subtract Rational Numbers	Multiply Rational Numbers	Divide Rational Numbers	Evaluate Numerical Bases	Product Property	Quotient Property	Scientific Notation	Add	Subtract	Multiply Mon by Mon	Multiply Poly by Mon	Multiply Bin by Bin	Divide Mon by Mon	Divide Poly by Mon									

Rating Scale

Activity: Problem Solving **Date:**
Course: Mathematics 10

Student	Description of Activity Component	Scale Points				
	1. Understands the Problem					
	2. Devises a Plan					
	3. Executes the Plan					
	4. Reflects					
	5.					
	1.					
	2.					
	3.					
	4.					
	5.					
	1.					
	2.					
	3.					
	4.					
	5.					
	1.					
	2.					
	3.					
	4.					
	5.					

Rating Scale

Activity: **Date:**
Course:

Student	Description of Activity Component	Scale Points				
	1.					
	2.					
	3.					
	4.					
	5.					
	1.					
	2.					
	3.					
	4.					
	5.					
	1.					
	2.					
	3.					
	4.					
	5.					
	1.					
	2.					
	3.					
	4.					
	5.					

GRADE: Mathematics 10 **TOPIC:** Linear Functions **DATE:**

A - able **U - unable**	Define	Identify	Generate Examples	Interpret	Graph			COMMENTS

GRADE: **TOPIC:** **DATE:**

								COMMENTS

GRADE: **DATE:**

TOPIC: Communicating Mathematically (oral and written)

	Clarify	Accuracy	Articulation	Proper Symbols	Proper Terms					

GRADE: **TOPIC:** **DATE:**

Observational Rating Scale

Group Work

Topic:

The student:	**Seldom - 1 Always - 4**	**Comments**
1. Volunteers information or ideas		
2. Shows willingness to listen		
3. Asks good questions		
4. Considers facts		
5. Shows respect for others		
6. Supports ideas with facts		

Teacher Notes:

	Seldom - 1 Always - 4	**Comments**

Teacher Notes:

Grade: Mathematics 10
Topic: Budget
Date: June 3

Student	**Rating** 1 - Fair 2 - Good 3 - Very Good 4 - Excellent	**Comments**
Corry	2	Understands the concept of budgeting but has difficulty working with percent when calculating portion of total income spent on categories.
Melissa	4	Fully understands all aspects of budgeting. Is able to plan, display, adjust and interpret a budget.

Grade:
Topic:
Date:

Student	**Rating** 1 - Fair 2 - Good 3 - Very Good 4 - Excellent	**Comments**

Anecdotal Records

Grade: Mathematics 10 **Date:** May 20-24
Activity: Angles and Polygons

Tom Is unable to identify vertically opposite angles in a diagram.				Jim Uses a variety of methods when calculating the measure of the angles and sides of a right triangle.
		Lori Works quickly and accurately when calculating the measure of complementary and supplementary angles.		
				Donna Confuses a central angle with an interior angle of a polygon.
Patti Can not remember and identify the various kinds of angle.		Brent Uses incorrect Trig. ratio when solving word problems.		
			Jason Has difficulty identifying the hypotenuse in a right triangle when the triangle is rotated.	
	Shannon Recognizes parallelograms but is unable to determine if one parallelogram has the properties of another.			

Anecdotal Records

Grade: **Date:**
Activity:

Individual Group Evaluation Form

We will stop once weekly to hold discussion in your groups about the process of working together. This is a time to consider how you feel and what you think about working in your group. Thinking about the process of working together helps people to recognize strengths and to become aware of ways in which we can improve our working relationships.

Please answer the following three questions on your own. Then use the three questions and your response as the basis for discussion within your group.

1. How do you feel about your participation as a member of your group at this time?

   Please circle:

   Very Satisfied   Quite Satisfied   Somewhat Dissatisfied   Quite Dissatisfied

   Please comment on why you checked where you did:

   ____

   ____

   ____

2. How do you feel about the productivity of your group at this time?

   Please circle:

   Very Satisfied   Quite Satisfied   Somewhat Dissatisfied   Quite Dissatisfied

   Please comment on why you checked where you did:

   ____

   ____

   ____

3. What things might I, or we, do to improve our group functioning as we continue to work?

   ____

   ____

   ____

Clarke, J., Wideman, R., Eadie, S. (1990). *Together we learn: Cooperative small group learning.* Scarborough: Prentice-Hall Canada Inc. p. 106. Reproduced with permission.

Observation Form

Place check marks in the appropriate boxes as you watch and listen. You might also want to record a few examples of what group members do or say.

	Asking Questions	**Seeking Information and Opinions**	**Responding to Ideas**	**Acknowledging Contributions**
Group 1				
Group 2				
Group 3				
Group 4				
Group 5				

Date ______________________

Clarke, J., Wideman, R., Eadie, S. (1990). *Together we learn: Cooperative small group learning.* Scarborough: Prentice-Hall Canada Inc. p. 131. Reproduced with permission.

Sample Criteria Checklist Developed by the Teacher and Students for Summative Peer and Teacher Evaluation

Criteria for Effective Group Presentation to Review Concepts in Trigonometry

The group:	Not at all				Thoroughly
! appeared prepared and organized.	__	__	__	__	__
! was knowledgeable about its section.	__	__	__	__	__
! worked together as a group.	__	__	__	__	__
! encouraged active participation from the class.	__	__	__	__	__
! demonstrated patience and helpfulness.	__	__	__	__	__
! used a variety of teaching techniques.	__	__	__	__	__

One part of the presentation which was particularly helpful (and why):

One suggestion for improvement:

Clarke, J., Wideman, R., Eadie, S. (1990). *Together we learn: Cooperative small group learning.* Scarborough: Prentice-Hall Canada Inc. p. 138. Adapted.

Sample Evaluation of Another Group's Presentation

! How did the group capture your interest in the topic?

! Describe three things you learned about the topic from the presentation.

! Describe one thing about the presentation that you thought was creative or imaginative.

! Make one suggestion which you think would strengthen the effectiveness of the presentation.

! Was there anything about the presentation that makes you interested in learning more about the topic? If so, please indicate what it was.

Names ____

Clarke, J., Wideman, R., Eadie, S. (1990). *Together we learn: Cooperative small group learning.* Scarborough: Prentice-Hall Canada Inc. p. 160. Reproduced with permission.

Unit Planning

There are many ways to plan a unit. The following unit planning guide is not prescriptive. Its intent is to offer some suggestions to teachers so that they may plan for the maximum benefits for their students. In any case, all planning should include expected learning outcomes, instructional strategies, and evaluation strategies.

Unit Planning Guide

1. Determine the sequence of the concepts that are to be developed.

2. Examine the Foundational Objectives with the subsequent learning objectives. Include the development of the Common Essential Learnings.

3. Determine the prerequisite skills required.

4. Identify the print and non-print resources to meet the needs of the students. Consider community resources.

5. Develop activities that are appropriate for the objectives.

6. Consider a variety of instructional strategies and methods for the activities. Select those that are most appropriate in meeting the objectives and in meeting the learning styles and needs of the students.

7. Determine the organizational structures that support the instructional methods and activities to be used.

8. Consider how the activity might be linked to other areas of study. Modify the activity to strengthen the connections.

9. Analyze how the Common Essential Learnings can be developed within the activities of each lesson.

10. Consider the special initiatives of Gender Equity, Indian and Métis perspective, and Agriculture in the Classroom. How can they be stressed during the unit?

11. Adapt for individual differences whether it be curriculum topics and materials, instruction, or environment.

12. Select student evaluation strategies. Assistance on this aspect of planning is available elsewhere in this guide and in *Student Evaluation: A Teacher Handbook* (1991). Just as a variety of activities should be chosen to accomplish the objectives, a variety of evaluation strategies should be employed so that all aspects of learning can be assessed.

Model Unit: Consumer Mathematics (Income and Budget)

Unit Overview

This model unit may be taught at any time. There are many ways to approach this unit. A problem solving approach is the one chosen here with emphasis placed on developing estimation, approximation, mental calculations, and comparison skills. It is hoped that this unit will give teachers possible ideas concerning instructional approaches, evaluation methods, the Common Essential Learnings, the Adaptive Dimension, and the other Core Curriculum initiatives.

In this model unit, students will explore the concepts of income and budget in Consumer Mathematics. This unit offers opportunities for the use and development of vocabulary and mathematical skills as students extend their knowledge and understanding about these two concepts. Attention is given to the connection between problem solving in the real world and the mathematics taught in school.

Income and budget are two key concepts that every consumer is faced with on a daily basis. Students must be able to apply consumer skills and many problem-solving strategies if they wish to cope with every day life. In this unit, students will learn: to use data to solve multi-step problems in which they compute total earnings involving overtime and commission as well as adjust budgeted amounts to meet unexpected expenses; to use estimation and mental calculations to check the reasonableness of answers and simplify calculations; to choose the correct operation to find hourly wages and expenditures; to use guess and check; to work backward to find amounts budgeted; to organize information to solve problems; and, to use critical thinking skills to compare and contrast various problem solving strategies and to determine if it is possible to adjust a budget to meet a changing lifestyle.

The sample lessons that follow and the suggested resources and activities serve as examples. Review the guidelines for unit planning which appear in this guide. All sample lessons should be adapted to accommodate students' needs, interests, strengths, and experiences.

A flexible time frame has been provided for this unit. It is recommended that 10 hours of instructional time be spent on this unit. One class period is equivalent to approximately 60 minutes. The actual amount of time spent on each lesson will be determined by the students' interest and background, the availability of resources, and the instructional strategies employed.

A classroom display of resources on income and budget may be used to stimulate interest and a thoughtful discussion of these concepts.

Note: Materials, resource person(s), and student assignments should be arranged well in advance of the beginning of this unit.

Foundational Objectives

- To apply simple mathematics to assist in the calculations and estimation of income and expenses.
- To develop a budget to guide current and future planning.
- To strengthen students' knowledge and understanding of **how** to compute, measure, estimate, and interpret numerical data, **when** to apply these skills and techniques, and **why** these processes apply within the study of income and budget. (NUM)
- To enable students to understand and use vocabulary related to income and budget. (COM)
- To support students in treating themselves, others and the environment with respect. (PSVS)
- To promote both intuitive, imaginative thought and the ability to evaluate ideas, processes, experiences, and objects related to income and budget. (CCT)

The selection of C.E.L.s for emphasis in this unit does not preclude the development of other C.E.L.s.

Detailed Learning Objectives

- To calculate weekly gross wages involving regular pay, overtime pay, and piecework earnings.
- To calculate earnings for straight commission or base wage plus commission.

- To determine the difference between gross pay and net pay.
- To calculate weekly, monthly, and yearly net pay.
- To define and explain the purpose of a budget.
- To determine and calculate the monthly fixed expenditures.
- To investigate the guidelines in developing a budget.
- To plan a budget and compare it to the percentages allotted to various categories stated by various institutions.
- To calculate the portion of total income spent on each category using percents.
- To draw graphs of budget figures (including circle graphs) using appropriate software.
- To calculate the actual amount of money to be spent on each category using the predetermined percentages.
- To adjust a budget to changes in expenses.

Other materials

The following materials are some that might be employed in this unit. New resources are being developed, and could be used in place of those in this list, as they become available.

- *Math Matters 10*, Nelson
- *Consumer and Career Mathematics*, Gage
- *Applied Mathematics*, McGraw-Hill Ryerson Limited
- *Mathematics For The Informed Consumer*, Gage
- *Mathematics for a Modern World Book 2*, Gage
- *Mathematics for a Modern World Book 3*, Gage
- *Consumer Mathematics*, Houghton Mifflin
- *Mathematics For Business*, Gage
- *The Budget Book*, Credit Union
- *The Spending Planner*, Wellness and Health Promotions Branch, Saskatchewan Health
- *Source Deductions*, Revenue Canada

Lesson Plan #1

Concept: Income

Objective(s): To calculate weekly gross wages involving regular pay, overtime pay, and piecework earnings.

Prerequisites: vocabulary - wages, salary, piecework, commission

Resources: *Consumer and Career Mathematics*
Applied Mathematics 10
Mathematics and the Modern World Book 2
Mathematics and the Modern World Book 3
Mathematics For Business
Consumer Mathematics
Mathematics For The Informed Consumer
Newspaper(s)
Newer resources as they become available

Time: 1 to 2 periods

Instructional Methods/Activities:

1. Have students begin the unit by noting questions they have about income and budget in a journal writing. Then, at the end of the unit they could answer their own questions in a journal writing.

2. Have students read the want ads for jobs that indicate wages. Saturday newspapers usually carry more of these. They can make a display of their findings.

3. Students who have part-time jobs may wish to discuss their pay scheme such as wages, salaries, or commissions. This will make the class discussion more meaningful as they are drawing on first-hand experiences. When students have the opportunity to talk about what they are learning, this allows for individual understanding and application. (COM)

4. Have the students work in small groups to discuss the following situation: You own your own travel agency and you must decide whether to pay your employees a salary, an hourly wage, or a salary (or wage) plus commission. A recorder and reporter should be appointed in each group. Each member of the group should be encouraged to speak, expressing thoughts and feelings. Students then report back to the large group. Class discussion then follows. Encourage students to follow the problem solving model where they identify the problem, list possible solutions, list consequences of suggested solutions, and select the best solution. Refer to the **Instructional Approaches** for further ideas on cooperative learning.

5. Why is it important to calculate weekly or even monthly earnings? Have students calculate weekly wages involving regular pay and overtime pay using the examples from the want ads. Encourage students to round off and estimate as a check for the reasonableness of their answers. Examples:

job: cashier
hours worked: 22
hourly rate: $6.90

job: gas jockey
hours worked: 28
hourly rate: $5.25

The memory key on the calculator is very useful in calculating the total pay involving regular and overtime pay. Use a problem solving approach to determine gross pay for a week that involves overtime. Students may use a variety of acceptable approaches.

Examples:

job: clerk
hours worked: 43
hourly rate: $5.10
overtime rate: time and a half

job: lifeguard
hours worked: 45
hourly rate: $7.00
overtime rate: double time

a) Define the problem: how to calculate gross pay.
b) Devise a plan:
calculate regular pay
calculate overtime rate
calculate overtime pay
calculate gross pay by adding regular pay to the overtime pay
c) Carry out the plan: regular pay - $5.10 X 40 = $204.00
overtime rate - $5.10 X 1.5 = $7.65
overtime pay - $7.65 X 3 = $22.95
gross pay - $204.00 + $22.95 = $226.95
d) Reflect back:
use estimation and mental calculations
estimate regular pay - 40 X $5.00 = $200.00
estimate overtime pay - 3 X $7.50 = $ 22.50
Total: $222.50
Therefore, $226.95 is a reasonable amount.

6. Investigate jobs that work by piece. Have students brainstorm for possible jobs that work by piece. Divide students and jobs up into groups and research each job. Report on the job, the amount of money paid, and the time it takes to complete the job.

7. Students evaluate working by piece stating the advantages and disadvantages. Why? Would you prefer to work by piecework, salary, or by a wage? Why? Which would you prefer as an employer?

8. Encourage students to evaluate and draw conclusions on the various jobs found in the want ads. Which jobs do they prefer? Why? What should one consider when making a decision concerning a job? (wages, hours, type of work, location, transportation, etc.)

9. Follow up by assigning questions on calculating weekly gross wages involving regular pay, overtime pay, and piecework.

Adaptation: Students may also consider calculating gross wages by including tips. They may consider paying a percentage of their earned tips to other employees that assist them in their job; e.g., a waiter decides to give 25% of his/her tips to the busboy.

Evaluation: Rating Scale (See assessment templates re: Problem Solving.)
Checklist
Peer and Self-Assessment (See assessment templates re: Individual Group Evaluation form.)
Performance Test (See the example with Lesson #5.)

Lesson Plan #2

Concept: Income

Objective(s): To calculate earnings for straight commission or base wage plus commission.

Prerequisites: vocabulary - commission, base wage percent

Time: 1 to 2 periods

Resources: *Applied Mathematics 10*
Consumer and Career Mathematics
Mathematics and the Modern World Book 2
Mathematics and the Modern World Book 3
Consumer Mathematics
Mathematics For The Informed Consumer
Newspaper(s)
Community employer
Newer resources as they become available

Instructional Methods/Activities:

1. In pairs, students find jobs in the want ads that include commission. What are some implications for working strictly on a commission basis? What factors would determine the items you would sell? Would you rather work on a straight commission basis or a salary and a commission? Explain. Would your answer be the same if you were an employer? Explain. (CCT)

2. Review how to calculate a percent of a number. How do you calculate commission?

3. Investigate jobs in the community that involve commission, if any. Prepare a written report on the job and the rate of commission received. Comment on the advantages and disadvantages for the employee and the employer when paid by commission. Students should either visit a employer or invite one into the class to discuss why a commission method was chosen, how it works, its advantages, and its disadvantages.

4. Students should work through examples of working on a commission or a base wage plus a commission. Encourage students to use a variety of strategies in determining the commission.

   Examples:

   A salesperson from a furniture store receives a 9% commission. How much money will the salesperson earn after selling $1200.00 worth of furniture?

a) Define the problem: how to calculate commission
b) Devise a plan: multiply the sales by the commission rate
c) Carry out the plan: .09 X $1200.00 = $108.00
d) Reflect back: use mental arithmetic
   9% = 10% - 1% so 10% of $1200.00 = $120.00 and 1% of $1200.00 = $12.00 so commission is $120.00 - $12.00 = $108.00

Jim works part time in the clothing department in a large store. He is paid a salary of $150.00 a month for the 4 Saturdays he works plus a 3% commission on his total sales. How much does Jim earn for the month of May if his sales totalled $6,230.00?

* Use the memory feature on the calculator to calculate total earnings.

Adaptation: You are offered two sales positions; one offers $500.00 per month plus 3% commission on all sales over $1 000 while the other offers a straight commission of 10%. What amount of sales would you need in one month to earn more selling on straight commission? Which job would you take? Why?

Evaluation: Observation Checklist
Performance Test (See the example with Lesson #5.)
Written Report (Example below.)

Rating Scale for Written Report	1 = poor				5 = good
Introduction	1	2	3	4	5
Statement of the Purpose of the Report	1	2	3	4	5
Coherent and Logical Development	1	2	3	4	5
Adequate Number of Examples	1	2	3	4	5
Good Mechanics in Writing	1	2	3	4	5
Good Mechanics in Mathematics	1	2	3	4	5
Originality	1	2	3	4	5
Knowledge of Topic	1	2	3	4	5
Incorporation of Specific Vocabulary	1	2	3	4	5
Conclusion (Summary)	1	2	3	4	5

Lesson Plan #3

Concept: Income

Objectives: To determine the difference between gross pay and net pay.
To calculate weekly, monthly, and yearly net pay.

Prerequisites: vocabulary - net pay and gross pay

Resources: *Applied Mathematics 10*
Consumer and Career Mathematics
Mathematics and the Modern World Book 2
Mathematics and the Modern World Book 3
Source Deductions
Community employers
Parents/guardians
Newer resources as they become available

Time: 1 to 2 periods

Instructional Methods/Activities:

1. Pose the problem: Sheila has taken a job at McDonalds starting at $5.50 per hour. She plans to work 25 hours a week and so she is looking forward to a wage of $137.50 each week. Will Sheila actually receive a cheque for $137.50 each week? What are some factors that must be considered in deciding how much pay she will receive? What kind of tax information might be useful in finding out the actual amount of the cheque? How can Sheila find out what her take-home pay will be? Approximately how much of a person's wages or salary do you think is withheld? How much money would Sheila have to earn to take home this amount of money?

2. What is the difference between gross and net pay? Discuss an earning statement and examine the various types of deductions. Get the students to inquire about other deductions by asking their parents/guardians or using the learning resource centre.

3. In groups, students select an employer in the community and inquire about the various deductions that he/she deducts from an employee's gross income. Students prepare a set of questions that they will ask which in turn are approved by the teacher; e.g., What are some fringe benefits that are offered? How much do these cost the employer? What other deductions are subtracted from the gross pay? Students must present a brief oral report to the class. (COM)

4. Supply students with sample earning statements and have them check the computations. Students complete a pay slip showing the deductions for income tax, CPP and UI (tables are available from your District Taxation Office) and calculate the take home pay.

5. Students determine their weekly net incomes being sure to include allowances and wages from part-time jobs. They will have to determine what deductions are included with their part-time jobs. They can then calculate their monthly and yearly net pay.

Adaptations: Students gather information about pension plans and report on their findings to the class.
Students can determine what percent the net pay is compared to the gross pay.
Is the percent of the take home pay (net) smaller or larger as your gross pay increases?

Evaluation: Oral Report
Performance Test (See the example with Lesson #5.)

Rating Scale for Oral Report	1 = poor				5 = good
I. Concept					
! poses the problem, situation	1	2	3	4	5
! demonstrates mathematical understanding through use of examples	1	2	3	4	5
! uses mathematical vocabulary correctly	1	2	3	4	5
II. Presentation					
! speaks to whole group	1	2	3	4	5
! speaks in a clear and steady voice	1	2	3	4	5
! uses related visuals (charts, overheads, etc.)	1	2	3	4	5
! uses appropriate positioning and posture	1	2	3	4	5
! presents with originality	1	2	3	4	5

Lesson Plan #4

Concept: Budget

Objectives: To define and explain the purpose of a budget.
To determine and calculate the monthly fixed expenditures.
To investigate the guidelines in developing a budget.

Prerequisites: percent

Resources: *Consumer and Career Mathematics*
Applied Mathematics 10
Math Matters 10
Mathematics For Business
The Spending Planner
The Budget Book
Newspapers and magazines
Community financial institutions
Newer resources as they become available

Time: 2 periods

Instructional Methods/Activities:

1. Individually write about "What is a budget?" (5 minutes). With a partner and then in groups of four, students discuss the following questions. What is a budget? What is it used for? Why is it used? Who uses a budget? They must also find examples of graphs in newspapers and magazines that illustrate budgets. They should reflect on the previous questions in light of their examples. Each group then presents its findings to the entire class. The class collaboratively arrives at possible answers to each question by evaluating the presented responses. This process (1-2-4: individual, pair and group of four) allows for individual reflection prior to discussion. (COM)

2. Students classify their expenditures (fixed and variable) into categories of their choice; e.g., savings, food, clothing, entertainment, transportation, and miscellaneous. How much might you spend each month on each of these categories? Does it seem reasonable? How can you determine the amount you spend on the average each week? Students should record their expenses for one week on an expense sheet with their chosen categories.

3. Find the average monthly expenses when given the monthly expenses for each category. Calculating the average expenses over several months for each category will give students an idea of how they spend their money. This will assist in anticipating how much they need for each category and in planning the spending of their money.

4. If students have not already done so they should calculate or estimate their weekly net incomes. How does this compare to their expenses? What percentage is each category of their net income? Why is it easier to compare by using percentages?

5. What percentage amounts are acceptable for each category when budgeting your money? Are they the same for everyone? Where could these guidelines be found? Students should visit various financial institutions to investigate the guidelines for developing a budget; e.g., Credit Union has the booklet *The Budget Book*.

Adaptation: If parents approve, students may wish to examine and categorize their family's expenditures.

Evaluation: Rating Scale and Anecdotal Records (See assessment templates)
Performance Test (See the example with Lesson #5.)
Peer and Self-Assessment

Self-Assessment could be done by having students keep a journal in which their entry would focus on their knowledge, attitude, and skill development. In this lesson, they would respond by being asked if they knew what a budget was, and why budgets are kept; if they felt they could determine what fixed expenditures were, and if they might be successful in developing a budget on their own.

Self-Assessment: Budgeting	**Response**		
1. I can explain what a budget is to others	Yes	Somewhat	No
2. I know why a budget is necessary sometimes.	Yes	Somewhat	No
3. I know what "fixed' expenditures are.	Yes	Somewhat	No
4. I can calculate the total amount of fixed expenditures for a time period.	Yes	Somewhat	No
5. I feel confident that I could develop a budget given some guidelines.	Yes	Somewhat	No
6. I need to know more about budgets.	Yes	Somewhat	No

Lesson Plan #5

Concept: Budget

Objectives: To investigate the guidelines in developing a budget.
To plan a budget and compare it to the percentages allotted to various categories as stated by financial institutions.
To calculate the portion of total income spent on each category using percents.
To draw graphs of budget figures (including circle graphs) using appropriate software.

Prerequisites: percent, measuring angles with a protractor, and constructing circle graphs.

Resources: *Consumer and Career Mathematics*
Math Matters 10
Mathematics For Business
The Spending Planner
The Budget Book
Newer resources as they become available

Time: 2 to 3 periods

Instructional Methods/Activities:

1. Many people who spend without a plan find themselves always in debt and unable to save. By planning a budget, you can decide whether optional purchases fit into your spending plan and how much you need to save in order to meet payments or unexpected expenses. Review the advantages of planning and following a budget.

2. Discuss the percentages allotted to various categories used to develop a budget. The booklet, *The Budget Book*, distributed by the Credit Union states the percentage of take home pay that might be allocated for various categories for the average Canadian family. They include:

   savings 5% - 10%
   food 18% - 30%
   clothing 8% - 15%
   health and miscellaneous 14% - 30%
   housing 18% - 30%
   utilities 5% - 9%
   transportation 10% - 15%

   Are the percentages relevant to an average student? Why or Why not? Will the percentages change in each category as a student gets older? Where might the changes occur? (You may want the students to note that the minimum % is 78% while the maximum % total is 139% in the suggested budget allocations.)

3. Divide the class into groups of three and have each group appoint a recorder. Give each group a large sheet of paper and a marker. Have each group make a circle graph in which they identify the categories in a teenager's spending plan along with the percentages of total income spent on each category. Only estimated percentages are required at this point. Display the pie graphs on the bulletin board. Each group should give a brief presentation of its circle graph explaining the reasons for the categories and percentages. How do they compare to the percentages for an average Canadian family mentioned previously? How do they compare with one another?

4. Students should use their figures for net income and expenses from the previous lessons to plan a realistic budget for themselves. Be sure the money in each category does not exceed the net income. In order to compare their budgets with one another, have the students calculate the percentage that comprises their net income in each category.

5. Students should display their budgets on a circle graph. This will enable them to easily compare the amounts spent on different items. Use a calculator to determine the number of degrees needed to construct a central angle for each section of the circle graph. Students may use a protractor to measure the central angle. How do the categories compare? Are any changes needed?

6. A follow-up activity involves students working in groups of 2 or 3 to plan a budget for a shared-accommodations situation (or school trip). They must research the fixed and variable expenses involved in such a circumstance. They must then determine the amount of money at their disposal (net income) and budget according to their predetermined categories. Their budget must include a circle graph displaying the categories and percentages. Compare the circle graphs with others in the class. How are they the same? How are they different? Why? (CCT)

Adaptation: A percent of a circle protractor can be useful in helping students draw circle graphs as they only need to calculate percents. They do not need to calculate the central angle in degrees.

Make a circular protractor calculated in percents:
0% --> 0°
50%--> 180°
100%--> 360°
calculate in 5% patches.

Evaluation: Peer and Self-Assessment
Observation Checklist
Performance Test (See the example below.)

Performance Test
(could be based on 6 above)

Students could be given some information about costs of accommodation and transportation, and be asked to plan a budget for a sports team to go to a tournament, or a band to take a trip to a competition. The number of students going would also be given.

Scoring Rubric

0 No attempt.
1 Copied information.
2 Some start on budget, given costs included.
3 Given cost, student budgeted correctly. Some indication of variable costs included.
4 Given costs, student budgeted correctly; many variable costs included, and budgeted correctly.
5. All reasonable costs, both fixed and varied, included; correctly budgeted and displayed on a circle graph.

Variable costs: include items such as:

1) food
2) snacks
3) souvenirs
4) admission for sight-seeing, entertainment
5) unforeseen expenditures (for example, a new reed for wind instrument, etc.)

Lesson Plan #6

Concept: Budget

Objectives: To calculate the actual amount of money to be spent on each category using the predetermined percentages.
To adjust a budget to changes in expenses.

Prerequisites: percent

Resources: *Consumer and Career Mathematics*
The Spending Planner
The Budget Book
Newer resources as they become available

Time: 1 period

Instructional Methods/Activities:

1. Review suggested percentages recommended by the Credit Union or any other financial institution. Use either the suggested guidelines or the ones arrived at in the previous lesson to calculate the actual dollars spent for each category; e.g., How much of a $90.00 weekly income could be spent on clothing if 15% is budgeted for clothing?

2. Are the actual amounts reasonable for each category? Keep track of your expenses for a week and then compare the totals in each category to the budgeted amounts. Did you stay within your budget? If not, is it possible that you under or over budgeted for certain categories? How could you accommodate unforeseen expenses? Is your budget reasonable? Should you cut back on a category next week? How might you adjust your budget? Students should discuss the difference between needs and wants. What impact does consumer spending have on the environment?

3. A budget may have to be adjusted. Why? What are some possible items that might force you to rethink your budget? e.g., buying a car, a school jacket, trip, etc. What categories could be adjusted to cover unforeseen expenses? Have students brainstorm for possible solutions; e.g., Jill's net income is $380.00 per month and she has budgeted her money accordingly: clothing - 30%, entertainment - 15%, transportation - 15%, gifts - 10%, food - 15% and miscellaneous - 15%. Her car needs $189.00 worth of repairs. Will she have enough money to pay her bill if she takes the money budgeted for clothing, entertainment, and miscellaneous? How could the expenses be evenly distributed among the three categories?

4. Have students adjust their own budgets based on a new or unforeseen expense. For example, if their parents decide that they should pay something for room and board, how will this affect their budget? What would a fair payment be based on their income? Why? Students should discuss their thoughts and feelings for their decisions with a partner or with a small group.

Adaptation: If appropriate, students plan a budget for their family. (CELs)

Evaluation: Teacher Anecdotal Records (See assessment templates.)
Written Report
Performance Test (See the example with Lesson #5.)

Mathematics 10 Curriculum

Foundational Objectives - Mathematics 10

The Foundational Objectives describe the most important understandings and abilities which should be developed over the course of a unit or a year. They provide guidance to teachers in unit and yearly planning and should be within the range of abilities of the majority of students.

The Foundational Objectives form the basis for curriculum evaluation.

The following foundational objectives outline the important understandings to be developed in Mathematics 10.

1. To demonstrate the ability to solve linear equations and inequalities. (10 01 01)

2. To be aware that the graph of a first degree equation of two variables is a line and conversely that the equation of a line is a first degree equation of two variables. (10 02 01)

3. To produce graphs of linear equations and conversely, when given key information of the graph, find its equation. (10 02 02)

4. To use the knowledge of linear functions and equations to solve problems involving direct and partial variation. (10 02 03)

5. To use the concept of slope to measure the steepness of a line. (10 02 04)

6. To demonstrate the ability to work with arithmetic sequences. (10 02 05)

7. To apply simple mathematics to assist in the calculation and estimation of income and expenses and to develop a budget to guide current and future planning. (10 03 01)

8. To communicate a summary of financial projections in appropriate reports, tables, and graphs. (10 03 02)

9. To develop an informal understanding of the relationships between lines. (10 04 01)

10. To identify and apply common properties of triangles, special quadrilaterals, and n-gons. (10 05 01)

11. To apply the special quadrilaterals to real-world situations. (10 05 02)

12. To develop an understanding of Pythagoras' Theorem, the primary trigonometric ratios and their applications. (10 05 03)

13. To make the transition from arithmetic skills to algebraic skills. (10 06 01)

The development of the following foundational objectives for the Common Essential Learnings (C.E.L.s) does not preclude the development of other C.E.L.s objectives.

The Common Essential Learnings (C.E.L.s) are coded throughout this document as follows:

CCT - Critical and Creative Thinking
COM - Communication
IL - Independent Learning
NUM - Numeracy
PSVS - Personal and Social Values and Skills
TL - Technological Literacy

! To enable students to understand and use the vocabulary, structures, and forms of expressions which characterize mathematics. (COM)

! To strengthen students' understanding within mathematics through applying knowledge of numbers and their relationships. (NUM)

! To strengthen students' knowledge and understanding of **how** to compute, measure, estimate, and interpret numerical data, **when** to apply these skills and techniques, and **why** these processes apply within mathematics. (NUM)

! To support students in treating themselves, others, and the environment with respect. (PSVS)

! To develop students' abilities to access knowledge. (IL)

! To develop a contemporary view of technology. (TL)

! To promote both intuitive, imaginative thought and the ability to evaluate ideas, processes, experiences, and objects in meaningful contexts. (CCT)

Western Protocol - General Outcomes Grades 10 - 12

Saskatchewan Education, Training and Employment is part of a Western Protocol agreement to develop a common curriculum framework in several areas, including mathematics. The following General Outcomes are taken from the Western Protocol, *Common Curriculum Framework for Mathematics* (draft, March 1995) and are presented here for your awareness.

Numbers
Number Concepts

- Explain and illustrate the structure and the interrelationship of the sets of numbers within a number system.
- Describe and apply arrays in the formulation and solution of problems.
- Describe, apply, and transform arrays in the formulation and solution of problems.

Number Operations

- Apply the basic operations singly or in combination, (and their interrelationship) on any real number and illustrate the use of these operations in solving real-world problems, such as budgeting.
- Describe and apply operations on arrays to solve problems, using spreadsheet technology as required.
- Describe and apply operations and transformations on arrays to solve problems, using spreadsheet technology as required.

Patterns and Relations
Patterns

- Represent by models, naturally-occurring data, using linear and nonlinear functions.
- Represent by models, those situations involving natural growth and decay.
- Represent by models, naturally-occurring data that are cyclical in character.

Variables and Equations

- Represent and analyze situations that involve variables, expressions, equations, and inequations.
- Connect and analyze functions and their graphs to solve problems.
- Translate between algebraic and coordinate relationships and the corresponding geometric shapes to solve problems.

Relations and Functions

- Translate between the various representations of discrete and continuous functions.
- Link models involving natural growth and decay to the exponential function.
- Link models involving data of a cyclical character to trigonometric functions.

Shape and Space
Measurement

- Solve problems using modern measuring devices such as electronic tuning forks, calipers, micrometer, electronic tape measures, and instrument-computer interfaces.
- Use the special properties of right triangles, including the sine, cosine, and tangent ratios, to solve problems involving more than one right triangle.
- Design an object to satisfy constraints such as maximum or minimum.
- Investigate the combination of vector quantities.
- Apply the sine and cosine laws to solve real-life problems involving general triangles.

3-D Objects and 2-D Shapes

- Justify congruence and similarity using transformations and coordinate geometry.
- Derive the properties that relate chords, tangents, and arcs and angles in circular and spherical geometry.
- Make and justify conjectures regarding the properties of a set of related figures.

Transformations

- Apply coordinate geometry and pattern recognition to predict the effects of combinations of transformations on 1-D and 2-D objects.
- Represent transformations numerically.

Statistics and Probability
Data Analysis

- Describe, implement, and analyze sampling procedures, and draw appropriate inferences from the data collected.
- Apply curve-fitting and correlation techniques to analyze experimental results.
- Analyze discrete and continuous data distributions to produce descriptions and inferences about populations and samples.

Chance and Uncertainty

- Use expected gains and losses to make and analyze decisions based on a single event.
- Model the probability of a compound event using the probability of simple events, and use these models to solve problems involving uncertainties.
- Solve problems based on the counting of sets or the combining of simpler probabilities.

Concept A: Linear Equations and Inequalities

Foundational Objective

! To demonstrate the ability to solve linear equations and inequalities. (10 01 01). Supported by the following learning objectives:

Objectives	Instructional Notes
A.1 To solve linear equations in one variable containing: b) variables on both sides c) parentheses d) fraction or decimal coefficients	This is a review of grade 9 material. Review and extend where necessary. Translation word problems may be used to introduce solving equations. A balance scale may also be used to reinforce the concept of equality. In small groups, students can create and solve their own equations. Each group can then exchange its set of equations with another group to solve. The groups then return their solved equations to be checked by the students. The equations can also be evaluated by the students according to the level of difficulty. (CCT) Students could also be given a completed question, where each step is on a separate slip of paper, and be asked to put them in the proper order. This reinforces the methods of solving an equation.
A.2 To solve a formula for an indicated variable.	Students generally have difficulty with this objective because of its abstractness. Review solving an equation with one variable before progressing on to formulas involving two or more variables.
A.3 To solve, graph, and verify linear inequalities of one variable.	Students should be aware that when multiplying or dividing by a negative value the direction of the inequality sign is reversed. Have students discover this by solving several inequalities and checking the results in the original inequality. They may also discover this by adding, subtracting, multiplying, and dividing an inequality by various values to determine if it remains true.

Examples/Activities

1) 2x + 7 = 5x - 29
2) (x - 11) = 2x + 7
3) 0.3y + 13 = 0.24y + 2
4) 2/5(z + 3) = 1/3(z - 5)
5) $\frac{x - 3}{6} - \frac{2x - 5}{8} = \frac{7x}{16}$

Adaptations

Change the roots in the solution set to integers. Enrich by solving equations with absolute value or rational expressions. Be sure students check for extraneous roots. Students may also be given the solution and it is up to them to create the equation.

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1) Find the amount of time in the formula d = rt, if d = 120 km and r = 30km/h. Solve for t in d=rt.
2) Solve for w in the formula p = 2l + 2w.

Solve for a variable in a formula that contains an exponent or radical; e.g.,

$$A = \pi r^2, \; F = \frac{mv^2}{r}, \; t = \sqrt{\frac{2d}{g}}$$

---

1) 5x - 34 > 2x + 14

2)

	add 2	subt 2	x by 6
3 < 9	5 < 11	11 < 7	18 < 54
True	True	True	True

x by -6	÷ by 3	÷ by -3
-18 < -54	1 < 3	-1 < -3
False	True	False

Therefore, when an inequality is multiplied or divided by a negative value, the inequality sign must be reversed.

3) -2y + 17 ≥ 31

Solve compound inequalities; e.g.,
1 < 3x - 5 < 10,
5x + 7 > 2x + 1 > x - 9,
x + 5 < 2x - 5 or
x - 4(x -1) < 2(4 - x)

You could also show students how they can transpose to make the coefficient of the variable positive.
ie. -2y + 17 ≥ 31
Add 2y. 17 ≥ 2y + 31
Subtract 31. - 14 ≥ 2y
-7 ≥ y
y ≤ -7

Concept A: Linear Equations and Inequalities

Objectives	Instructional Notes
A.4 To translate English phrases into mathematical terms and vice-versa.	Have students generate a list of English phrases that relate to real-world situations. Students then exchange lists so they can translate into mathematical symbols. Students may also create a sentence or an entire word problem based on a given expression or equation. Give students ample time to translate English phrases into mathematical expressions.
A.5 To solve real-world problems using various problem solving strategies. (CCT)	Students can generate and solve their own word problems that relate to real-world situations based on their own cultural backgrounds. Emphasis should be placed on practical situations and should encourage the use of different problem-solving strategies; e.g., using a diagram, table or list, guessing and checking, looking for a pattern, working backwards, or solving a related simpler problem, etc. **Avoid solving problems according to type.** Give students a wide variety of problems that may be solved in many different ways. The students can be encouraged to work in pairs, or in groups, and to compare their answers and method of solution with others in their group. The group work should emphasize the respectful treatment of each other. (PSVS) The teacher can ask for a solution from a group, and have group members explain how they arrived at the solution. The other groups could be asked if they agree with the solution, and why or why not. Other methods of solution can be solicited from the remaining groups, and similar discussion carried out.

Examples/Activities

1) five more than twice the number of boys.
2) 25n + 35(20 - n) = 640

1. At the end of October, Jim decides to attend volleyball camp the following July. If the camp costs $250.00 and he has already saved $50.00, how much money must he save each month?

   Many students will solve this problem by arithmetic:
   250 - 50 = 200;

   200/8 = 25. Stress the role of algebra in solving these simple problems; e.g., Let x = the amount which must be saved each month. 50 + 8x = 250

   Encourage students to think algebraically, and apply problem solving strategies to these problems.

2. If students are not able to create enough problems of various types, resource texts should be used as a further source of problems to examine. Try to choose a variety of problems from these resources. These might also be adapted to reflect local issues, concerns, or situations.

3. Another source of problems can be found in *The Mathematics Teacher*, the monthly journal of the National Council of Teachers of Mathematics (N.C.T.M) or in various Math contest papers.

Adaptations

The vocabulary involved when translating from English phrases to mathematical phrases can be quickly reviewed. Students might be asked to generate the vocabulary that could be used for +, -, x, ÷, .=, and so on, before they begin any translations. The teacher could introduce others, and ask the students what operation might be indicated. (COM)

Include extraneous information; e.g., Kathyrn made 20 L of car coolant that had a 75% antifreeze concentration. Last time she made the concentration 60%. How much water must she add to decrease the concentration to 50%? (NUM)

Concept B: Relations, Linear Functions, and Variation

Foundational Objectives

- To be aware that the graph of a first degree equation of two variables is a line, and conversely that the equation of a line is a first degree equation of two variables. (10 02 01). Supported by learning objectives 1, 2, 3, 4, 5, and 9.

- To produce graphs of linear equations, and conversely, when given key information of the graph, find its equation. (10 02 02). Supported by learning objectives 6, 7, 8, 10, 13, 14, 15, 16, 17, and 18.

- To use the knowledge of linear functions and equations to solve problems involving direct and partial variation. (10 02 03). Supported by learning objectives 19 to 23.

- To use the concept of slope to measure the steepness of a line. (10 02 04). Supported by learning objectives 11 and 12.

- To demonstrate the ability to work with arithmetic sequences. (10 02 05). Supported by learning objectives 24 to 27.

Objectives	Instructional Notes
Part I: Theory	
B.1 To define the following terms: relation, ordered pair, abscissa, ordinate.	Have students generate the definitions through guided questioning. This may serve to tie B.1 and B.2 together. Definitions might also be introduced using concept attainment.
B.2 To identify and express examples of relations in the real world.	Have students brainstorm for examples of relations around them. They should be aware of how the two things relate to each other in their examples. Draw examples from science, business, and everyday life.
B.3, B.4 To graph ordered pairs in the Cartesian coordinate plane, and to graph real-world relations in the Cartesian coordinate plane.	Students should be familiar with graphing ordered pairs in the Cartesian coordinate plane from the middle level. Use the example of latitude and longitude to show the real-life application of using coordinates; e.g., air traffic controllers direct airplanes by giving the location of planes using coordinates. (TL) Also, an atlas gives coordinates of cities such as C 4 which is similar to (3,4). Note that horizontal distance is given first in this system, but down is negative if you're using the letters at the top. Relate to seat games, like Battleship or BINGO tickets.

Examples/Activities **Adaptations**

Examples may include:

Cost of a combine in various years, total sales of a grocery store in each month of the year, amount of precipitation in each month of the year, earnings to the number of hours worked, distance travelled to the number of litres of gas consumed, value of a car to its age.

1) Plotting ordered pairs that create pictures, when the points are connected, is useful for motivating students. Students can create their own and then exchange listings of ordered pairs to their classmates.

2) Using a map of the world to show the locations of various cities and tourist areas will help students to see the application of using coordinates to locate points.

Concept B: Relations, Linear Functions, and Variation

Objectives	Instructional Notes
B.5 To read information from a graph. (NUM)	Students should be able to interpolate information between two given ordered pairs. They should also be able to extrapolate information by extending the line of a graph. Use data collected from natural and/or social sciences.
B.1.(b) To define the following terms: function, linear function, slope, x-intercept, y-intercept, ratio, proportion, direct variation, partial variation.	Make this enjoyable by using crossword puzzles or posters. Groups could be assigned a term to research in various sources and then present this term to the entire class. Students could be asked to generate examples illustrating some of these definitions.
B.6 To identify, graph, and interpret examples of linear functions describing real-world situations. (CCT)	Students should be able to distinguish between linear and non-linear functions. Concept attainment may be used to develop the concept of linear functions by giving examples and non-examples. They may review their examples of relation from objective B.2 and determine which are linear functions. Have the students graph the information for one of their examples. They should also determine the relationship that exists in their example.
B.7 To graph a linear function using a table of values.	The domain for linear functions is the set of real numbers unless restrictions are stated. Encourage students to choose the most appropriate values for the domain so that graphing the ordered pairs is done with ease. Be sure graphs are large enough so they are easy to read. Review solving a formula for an indicated letter, and encourage the students to solve the equation for y.
B.8 To determine if a relation is a function by employing the vertical line test.	Students should review the definition of function, and graph several so that the entire class can observe them. Some graphs should also be drawn that do not represent functions. Students should discuss the difference between the graphs of a function, and a non-function. The vertical line test may be developed by the students, or introduced by the teacher. Use the vertical line test to determine whether several graphs are functions or not. Have students check these results by using the definition of function.

Examples/Activities

Draw the graph of the relation representing earnings, if a person earns $5.00 an hour. This line should be drawn so that (1, $5) (2, $10) (3, $15) (4, $20), etc. are on the line.

Using the graph created in number 3 above, answer the following questions:

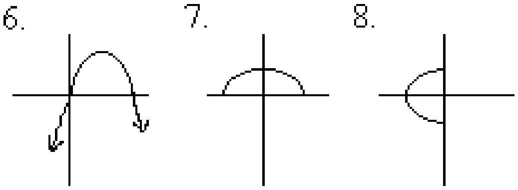
a) How much money is earned for 2.5 hours of work?
b) How many hours must a person work to earn $28.75?
c) How much money is earned for 8 hours of work?

1) total cost to the number of slurpees purchased
2) amount of allowance to a person's age
3) amount of daylight to the time of year

1) y = 2x + 1
2) y = **a**x - 4
3) x - 2y = 3

Do the following represent functions. Use the vertical line test.
Plot
1. {(2,6) (4,3)(3,5)(1,2)(0,5)}
2. {(3,2)(4,-1)(2,-3)(1,4)(3,-2)}
3. 2x - y = 5
4. x = 2
5. y = -1

6. 7. 8.



Adaptations

Graph a series of linear equations using a table of values. Place restrictions on the domain or range to create pictures; e.g., y = x + 3,
D = {x : 1 ≤ x ≤ 4, x å R}

These activities may be used to introduce the next topic; e.g., solve x - 2y = 3 for y-
Students will realize solving the equation for y is much easier than substituting values in 3x - 2y = 6, and solving for y after substituting each value in the table.

Concept B: Relations, Linear Functions, and Variation

Objectives	Instructional Notes
B.9 To solve equations in two variables, given the domain of one of the variables.	Students have already **graphed linear equations in two variables using a table of values in grade 9.** The students should be aware that restrictions are sometimes placed on one of the variables which will alter the graph of the linear equation.
B.10 To determine if an ordered pair is a solution to the linear equation.	Point out that the solution set for an equation consists of all the ordered pairs that lie on the graph of the equation. Substituting the coordinates of an ordered pair into the equation will determine if the ordered pair is a solution or not. Stress the use of parentheses when substituting in for the variables.
B.11 To calculate the slope of a line: a) graphically (m = rise/run) b) algebraically $m = \frac{y_2 - y_1}{x_2 - x_1}$ c) from the equation (y = mx+ b)	Have students suggest real-world examples of slope before they calculate slope. When do we use slope? Develop the concept of slope intuitively by relating it to steepness. Determine the slope of a line graphically by identifying two ordered pairs on the line. Do several examples and have the students derive the slope-point formula so they may calculate the slope of a line from just two ordered. Students can also be investigating lines that have a negative slope and positive slope. (Read up as positive, down as negative) Use a guided discovery approach to develop the relationship between the actual slope of a line and its equation in slope-intercept form.
B.12 To determine the slope of horizontal, vertical, parallel, and perpendicular lines.	Have students investigate the slope of horizontal lines and vertical lines both graphically and algebraically. Students should be able to identify lines that have positive, negative, zero, and undefined slopes. The slope of parallel lines can be investigated by first intuitively determining how the slopes of parallel lines compare. This can be accomplished by encouraging students to think of real-world examples of parallel lines. This same process can be duplicated for perpendicular lines.

Examples/Activities

Find a set of ordered pairs for each of the following:

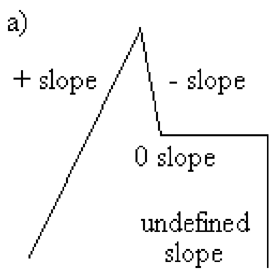
1) $x + y = 10 \text{ where } x \in \{-2, -1, 0, 1, 2, 3, 4\}$

2) $2x - y = 24 \text{ where } D = \{x \mid x \geq 0\}$

3) $x + 3y = 12 \text{ where } R = \{y \mid y \geq 0\}$

Which of the ordered pairs (2, -3) (4, 1) (5, 0) (4,3) satisfy the equation 3x - y = 9?

Real-world examples of slope include: mountains, roofs, ramps, planes taking off or landing, highways, and stairways.



b) Calculate the slope of the line segment joining:
(2, -3) and (5, 6)

C) Determine the slope in each case.
y = 2x - 5
3x - 2y = 6

Students could generate examples of parallel lines: girders in a building, lines in a parkade, fences, rows of crops, rungs on a ladder, streets, flight paths of the Snowbirds, and ramps to load cars onto a platform.

Students could be asked to give examples of perpendicular lines: rafters and wall studs, and side of a ladder and its rungs.

Adaptations

A true understanding of the fundamental issues involves congruent corresponding angles, similar triangles (AA), and the fact that corresponding sides of similar triangles are proportional. This proportion stated

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

, where the right side is equal to m.

Concept B: Relations, Linear Functions, and Variation

Objectives

Instructional Notes

B.12.(b)
To write linear equations in:
a) slope-intercept form
b) standard form

Why express linear equations in slope-intercept form? Students may need to review isolating a variable by solving a linear equation in one variable.

B.13 & B.14
To graph a linear equation in two variables using:
b) x and y intercepts
c) the slope and an ordered pair
d) the slope and y-intercept
(y = mx + b)

Students have been exposed to graphing linear equations in two variables using a table of values. How many points are necessary to determine the graph of a linear equation? Which coordinates are easiest to calculate? What is true for all x-intercepts? y-intercepts? How can a person find the x and y intercepts without graphing the linear equation? Students can practice their mental calculation skills by finding the x and y intercepts without paper and pencil. Students should also explore the general equations of horizontal and vertical lines.

How do the slopes between any two ordered pairs on the line compare to one another? Is it possible to locate another ordered pair on the line when given one ordered pair and the slope?
A guided discovery approach may be used to investigate the relationship between the slope of a line, y-intercept of a line, and the equation of the line. Students should be encouraged to work cooperatively in small groups in order to arrive at the relationships. (PSVS) Allow the students to report their findings to others in the class. Students may need to be reminded that the y-intercept represents an ordered pair and may be expressed as (0, b).

Examples/Activities	Adaptations

a) $2x + y - 1 = 0$
$x - 3y = 6$

b) $y = -4x + 7$

$$y = \frac{2x}{3} - 1$$

Graph each of the following.

1. $2x + y = 4$
2. $3x - 5y - 15 = 0$
3. a line having slope of 2/3 and passing through (1, -4)
   a line having a slope of -2 and passing through (2, 3)
4. a line having slope 4/5 and a y-intercept of -3
5. $y = x + 3$
6. $2x - y = 6$

Have students cover the x term (and coefficient) with their finger and solve for y. Repeat, covering the y term. Encourage them to solve these simple equations mentally.

Challenge the top students to figure out how to tell the slope just by looking at the equation in standard form: Ax + By = C. If they are unable to do this, have them complete a table using their answers.

Equation	Slope
2x + y = 1	-2
4x + y = 1	-4
x - 3y = 6	**b**
2x + 3y = 6	-**b**
.	.
.	.
.	.
Ax + By = C	-A/B

Find the slope and intercepts of:

$$\frac{x}{6} + \frac{6}{4} = 1$$

$$\frac{x}{3} + \frac{y}{2} = 1$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

Concept B: Relations, Linear Functions, and Variation

Objectives	Instructional Notes
B.15, B.16 To write the equation of a line when given: a) slope and y-intercept b) slope and one point on the line c) the graph of the line d) two points on the line	What information can be taken from the graphs of the previous examples in order to write the equation of the line? Students will come up with various suggestions. Allow them to use the information and then have them share their findings with other students. They should then reflect back and evaluate the information used to determine which information was the most advantageous. (COM) By this point, students should realize that they must find the slope of the line when writing the equation. Have students continue with the above problem solving approach to write the equation of the line when given two ordered pairs. Working with a partner may help to enhance the whole process.
B.17 To construct scatterplots from real-world data. (NUM)	These statistics topics fit well with the study of linear functions. Students can investigate various relationships and construct a scatterplot. A scatterplot is a graph representing the relationship between two variables that can be measured. Encourage students to draw conclusions about the scatterplots they constructed. The students can discuss and present reasons for their conclusions in small groups.
B.18 To interpret and critically analyze these scatterplots. (CCT)	Have students examine scattergrams they previously constructed. The equation of a line can be found so that the students can make predictions and estimations for unobserved values for one of the variables. Encourage students to write a paragraph about the information displayed in the graph along with inferences they believe are supported by their analysis of the data. (COM) Remind students that the line produced is not always accurate, or appropriate for all data; e.g., number of pages in a pocket novel to the price.

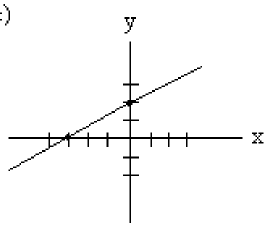
Examples/Activities

Write the equation for each of the following lines:

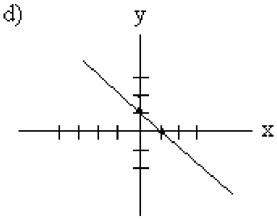
a) line having a slope of 3 and a y-intercept of 5

b) line having a slope of -2/3 and passing through (6, 1)

c)



d)



e) line passing through the ordered pairs (4, 6) and (-1, 7)

1) relationship between a car's age and the total mileage by collecting data from the school parking lot.

2) relationship between the number of points a player scored at a basketball game and the number of shots attempted.

3) relationship between the year and the length of the winning throw in discus at the Olympics since 1896.

Construct scattergrams for each of the following:

1) the relationship of distance to time for participants in a fitness walk. R = D/T

2) the relationship of the total cost of gas to the number of litres required to fill the tank. p=TC/L

Adaptations

Write the equations of horizontal and vertical lines. Investigate the properties and write the equations of parallel and perpendicular lines.

Expand on a general strategy for solving many (all) of these kinds of problems for students who may be experiencing difficulty. Stress that a line is fixed in a plane once one point and its slope are known. Hence its equation is determined. Thus to find its equation, regardless of what is given, students need to employ the following strategy:

1) What point shall I use? (often these are several)
2) What is the slope? (often the slope formula can be used, or parallel or perpendicular lines will help)

Can some of these scatterplots be approximately defined by a line? What would the equation of such a line be? How could you make use of this information to predict future outcomes of various examples?

Would you feel comfortable making a decision based on your prediction?

Three methods of determining the line to be drawn are as follows:

Method #1

Draw in a line so that the number of points above and below the line are equal.

Method #2

Arrange data, in order, into three equal groups according to values of the horizontal coordinates.

Find the summary point for each group based on the median x value and the median y-value.

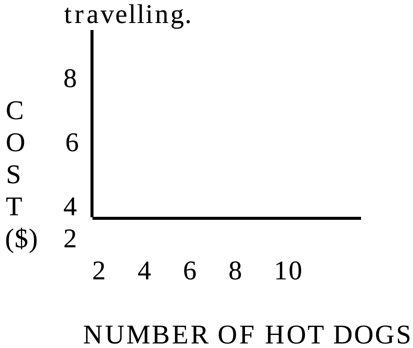
Draw a line through the summary points of the outer groups.

Objectives	Instructional Notes
Part II: Applications	
B.19 To identify, describe, and interpret examples of direct variation in real-world situations.	Students had been working with direct variation in the previous section. Have them make conclusions about their lines; e.g., begins at the origin and shows a constant growth. Refer to the students' graphs in the discussion and analysis of direct variation; especially when discussing the meaning of a constant. Ask students to find examples of direct variation from other classes they are taking, from magazines, and from newspapers.
B.20 To solve proportions involving direct variation.	Use the graphs of the above examples to relate the slope of the line to the constant of proportionality. Have students find the missing coordinates of an ordered pair using the slope (constant) and the graph of the line. Ask students how they might solve the same problem algebraically using an equation involving a ratio that relates x and y. Is it possible to solve the problem using a proportion consisting of two equal ratios? Students should be aware of both methods of solving the equations. Encourage students to approximate their answers in order to determine the reasonableness of their results. (NUM)

Examples/Activities

Students could be asked to generate direct variations such as the following and to answer questions based on these examples:

1) hours worked at a part-time job and weekly wages.
2) amount of interest earned and the amount of money invested.
3) distance travelled by motorcycle and the time spent travelling.



Find the value of y in the above direct variation.

Method # 1

Find the constant of proportionality. (cost/hot dog)

$k = y_1/x_1$, $k = 4/2$, $k = 2$

substitute $k = 2$ into the equation

$k = y_2/x_2$, $2 = y/8$, $y = 16$

Method #2

$$\frac{y_1}{x_1} = \frac{y_2}{x_2}, \quad \frac{4}{2} = \frac{y}{8}, \quad 2y = 32, \quad y = 16$$

Adaptations

Method #3

Follow the procedure in Method #2. Slide the line (L) one-third of the way to the middle summary point by:

1) finding the y-coordinate of the point on the line with the same x-coordinate as the middle summary point;
2) finding the vertical distance between the middle summary point and the line by subtracting y-values; and,
3) finding the coordinates of the point P one-third of the way from the line (L) to the middle summary point.

If available, Lotus 1, 2, 3 will calculate the slope and y intercept of the "line of best fit"; i.e., the regression line. It can also produce the scatter plot and the graph of the regression line. Students should note the use of the point-slope form to determine the line.

Solve direct variation questions that involve the square of one of the variables; e.g. $A \propto r^2$. (NUM)

Concept B: Relations, Linear Functions, and Variation

Objectives	Instructional Notes
B.21 To solve problems involving direct variation.	Be sure students can identify the two variables being related. They should organize their work and decide on the method they wish to use in solving the word problem.
B.22 To identify partial variation.	Illustrate several examples of partial variation. How does partial variation differ from direct variation? How are they alike? In particular, students should notice that the graph of a partial variation is a line that does not pass through the origin. There is a fixed part and a variable part. What is the slope of the line? How does the slope and y-intercept of a line compare to the fixed and variable part of a partial variation? Students could work cooperatively in groups to brainstorm for examples of partial variation from real-world situations. (PSVS)
B.23 To solve problems involving partial variation.	In groups, encourage students to create their own variation word problems that are of interest and relevance to them. They can create an answer key by solving the questions themselves. Groups can exchange questions which are in turn solved and returned to the group for verification. At this point, various questions can be discussed as a class for clarification. (COM) Ask students what was required to graph a linear equation. Is this information sufficient to write the equation of the line? Use the real-world, partial variation examples that students created in B.22 where the slope and y-intercept are easily identified. They should be able to see the relationship between the slope and y-intercept from the graph and the slope-intercept form of the equation. How would you write the equation of a line if given the slope and a point on the line? Students should be able to evaluate the given information and determine which form of a linear equation to use (slope-point or slope-intercept) in order to write the equation.

Examples/Activities	Adaptations
Calculate the solution in each of the following: 1. The amount of money earned on a job is directly proportional to the number of hours worked. If $44.00 is earned for 8 hours of work, how much is earned for 30 hours of work? 2. The amount of fertilizer needed for a lawn varies directly with the area of the lawn. If .5 kg of fertilizer is needed for 60 square meters of lawn, how much is needed for the Cain's lawn, which is rectangular in shape and measures 8 m by 16 m?	The price of a pizza varies directly with the square of its radius. If a 20 cm pizza cost $6.50, how much would a 30 cm pizza cost? Have students compare the calculated price, (above) with actual pizza parlor prices, for similar pizzas. Which is a better buy?
Draw graphs to represent each of the following: 1) car rentals: $25/d plus $0.12/km 2) plumber: $40.00 service charge plus $48.00/h 3) library loan charges: $1.00 overdue charge plus $0.25/day 4) bikes rentals: $10.00/day plus $2.00/day 5) pizza: $6.00 plus $0.80 for each additional topping 6) earning money: $10.00/day plus $0.05/flyer delivered (NUM)	Have students find other examples of partial variation. (IL)
Solve each of the following problems: 1) Lori receives a fixed salary of $200.00 per week plus a commission of 3% on all clothing sales. How much does she earn in a week if she sold $2500.00 of merchandise? 2) Troy has a chequing account that has an overdraft protection. The bank charges $5.00 plus 1.5% on the amount overdrawn. What are Troy's total charges if he was $800.00 overdrawn on his account?	Top students will notice that the slope intercept form is the same as the point slope form when the point given is (0, b).

Concept B: Relations, Linear Functions, and Variation

Objectives	Instructional Notes
B.24 To define, illustrate, and identify an arithmetic sequence.	Students could be given a set of questions involving number patterns, and asked to identify the next two numbers in the pattern. (Eg. 1, 2, 3, 5, 8, 13, 21, _, _) They could work on these patterns in pairs or in small groups. They could be instructed to determine if any of the number patterns showed a constant difference throughout. Several of these could be displayed, on the board, on an overhead, or on a flipchart. The class could then be informed that these represent arithmetic sequences and then be given the task of defining an arithmetic sequence. Once the arithmetic sequence has been defined, the class could be asked to brainstorm in small groups where arithmetic sequences are used in the real world. (eg. scores in hockey, baseball.) (COM) Given a series of sequences, the student should be asked to identify those which represent arithmetic sequences, and to explain why they are arithmetic sequences. Students should also be instructed to plot those sequences determined to be arithmetic sequences. This will allow them to connect visually an arithmetic sequence with a linear function. They could be asked to determine the slope and intercept of each and to write the equation representing each. Once students become familiar with the arithmetic sequence, they should be introduced to the basic definitions, and symbols used with arithmetic sequences: a, n, d, t_n, l and S_n. (COM)
B.25 To determine the n^{th} term of an arithmetic sequence.	Students could work in pairs on a series of questions involving arithmetic sequences. These questions could be a set of sequences where the first several terms are given, and where the students are asked to determine terms which would appear later in the sequence. These would be arranged so that they become correspondingly more difficult to determine. Each pair could be instructed to find the answers using arithmetic, and by plotting. Introduce the term t_n (or l in some texts), and instruct the students to determine a formula that might be used to find t_n for all cases. The students could work in groups of three or four to brainstorm through this activity. Student suggestions could be displayed on the board, overhead, or flipcharts. Each could be examined to determine if a counterexample is found. When students have agreed on the formula $t_n = a + (n-1)\,d$, an assignment can be given to the class.

Examples/Activities	Adaptations

1. Determine the next two terms in each of the following:

   a) 1, 3, 5 7, 9, _, _.
   b) 1, 4, 9, 16, 25, _, _.
   c) 3, 7, 11, 15, 19, _, _.
   d) 2, 4, 8, 16, 32, _, _.

2. Which of the following represent arithmetic sequences?

   a) 2, 5, 8, 11, 14, 17,.....
   b) 3, 6, 9, 15, 24, 39,.....
   c) 5, .5, .05, .005, .0005,.....
   d) 2, 7, 12, 17, 22, 27,.....
   e) f(x) = 2x+3, where x= 1, 2, 3, 4, 5,.....

3. Plot each of the arithmetic sequences identified in question 2.

   What is the slope and y-intercept of each?

   Write the equation for each.

1. Do each of the following using:

   i) arithmetic
   ii) plotting the sequence

   a) Identify the 6th term of 3, 7, 11, 15,.....
   b) Identify the 8th term of 1, 3, 5, 7,.....
   c) Identify the 9th term of 2, 8, 14, 20,.....
   d) Identify the 16th term of -2, 1, 4, 7,.....
   e) Identify the 37th term of 2, 3.5, 5, 6,5,.....
   f) Identify the 101st term of -3, -2.5, -2, -1.5,.....

2. Find t_n in each of the following:

   a) n=8, a=-2, d=4
   b) n=15, a=6 d=3.5
   c) n=22, a=2, d=-2
   d) n=37, a=-17, d=2.3

Students who are better in mathematics should be given questions where the unidentified variable is something other than t_n.
For example:
1. Determine a, if n=12, d, -3, and t_n=-28.
2. Determine the value of d, given that a=-5, n=21, and t_n=58

Objectives	Instructional Notes
	If students are not able to determine the formula, the teacher should provide a counterexample to the students' suggestion, and provide the formula for the students (with some examples).
B.26 To define arithmetic means, and to determine the required arithmetic means between given terms.	The definition of an arithmetic mean should be given to the students, through notes, reference to a text, and should be demonstrated in actual arithmetic sequences to the class. Students could be given an assignment to work on in pairs, or in small groups. For this assignment have the students determine the means of an arithmetic sequence, where the given information would consist of the first and last terms of the sequence, and the number of arithmetic means desired. See the examples on the next page. Further, the students should be instructed to determine these means by: i) arithmetic, and by ii) plotting the given points. It is helpful to use pairs or small groups when plotting points, so that students may check each others' plots to see if their points are correct. (PSVS)
B.27 To calculate the sum of an arithmetic series.	Students could work individually, in pairs, or in small groups, and be instructed to determine the sum of the natural (counting) numbers from 1 to 10. This could then be extended to determine the sum of the natural numbers from 1 to 100. By taking a selection of student answers and an explanation of how the students arrived at these answers, the teacher can direct the class toward the formula methods of finding the sum of a series. Definitions (e.g., series) and symbols used (e.g., S_n, a, n, d, l, t_n) should be introduced, and formulas given. Examples can be done by the teacher and/or students, and an assignment given.

Examples/Activities	Adaptations

1. Find the required arithmetic means for each of the following arithmetic sequences:

   a) 4 means, between 1 and 11.
   b) 5 means, between -3 and 3.
   c) 3 means, between -6 and 10.
   d) 5 means, between 8 and -1.
   e) 2 means, between 12 and 27.

1. Find the sum of each of the following:

   a) 1, 4, 7,.....25.
   b) -3, 1, 5,.....17.
   c) The first ten terms of 2, 5, 8,.....
   d) Where a = 5, d = -2, n = 18.
   e) Where a = -4, n = 14, l = 3.

Summation notation could also be introduced, with an explanation of how it is used.

1. Evaluate the following:

   a) $\sum_{n=1}^{8}(3n-2)$

   b) $\sum_{j=1}^{16}(4-2j)$

Concept C: Consumer Mathematics

Foundational Objectives

- To apply simple mathematics to assist in the calculation and estimation of income and expenses and to develop a budget to guide current and future planning. (10 03 01). Supported by learning objectives 1 to 9. (NUM)

- To communicate a summary of financial projections in appropriate reports, tables, and graphs. (10 03 02). Supported by learning objectives 10 to 12. (COM)

Objectives	Instructional Notes
C.1 To calculate weekly gross wages involving regular pay, overtime, pay, and piecework earnings.	When calculating wages, students should estimate to determine the reasonableness of their answers. Provide students with several examples of regular pay and overtime pay. Encourage the use of mental calculations and calculators when determining the overtime rate (time and a half, or double time). (NUM)
C.2 To calculate earnings for straight commission, or base wage plus commission.	Be sure students understand the difference between working on a commission and getting a commission plus a salary. Encourage the use of the calculator when calculating commission. A review of percent may be required at this time.
C.3 To determine the difference between gross pay and net pay.	Point out the difference between gross and net pay. Examine an earning statement to determine the types of deductions that an employer may deduct from an employee's gross income; e.g., C.P.P., U.I.C., Income tax, Pension plan, and any other deductions Note: both employer and the individual contribute to pension plans. Use the calculator to check the computations on an earning statement.
C.4 To calculate weekly, monthly, and yearly net pay.	After students have calculated their weekly net pay have them approximate their monthly and yearly net income.

Examples/Activities

Investigate jobs that work by piece; e.g., carpenters and mechanics. Report on several occupations that earn salary by piece. Be sure to give details of the job itself, the amount of money paid for each job, and the time it takes to complete certain jobs. Why is it important to calculate an average income for these occupations? For example: budgeting. (NUM)
Use the want ads to calculate weekly wages as well as monthly and yearly salaries.

In pairs, students find jobs that include commission in the want ads. Students should evaluate the job descriptions in order to decide which method of earnings is most advantageous.

If deemed appropriate, students may inquire about the deductions that are made on their parents' pay cheques. Research how some of these deductions are determined; e.g., C.P.P., U.I.C., and Income tax. What are some fringe benefits? How much do these cost the employer? (PSVS)

Have students determine their weekly net incomes being sure to include allowances and wages from part-time jobs. They should determine the deductions from their gross pay by checking with the *Source Deductions* from Revenue Canada. Monthly and yearly net incomes can then be calculated.

Adaptations

Teacher Notes

Concept C: Consumer Mathematics

Objectives	Instructional Notes
C.5 To define and explain the purpose of a budget.	Introduce the idea of a budget to the students. What is a budget? What is it used for? Why is it used? Who uses a budget? Students can keep track of their expenses for a week and then group them into categories; e.g., transportation, food, clothing, and entertainment. Have them estimate how much they would spend in one month. Supply students with figures for various months so they can determine average monthly expenses. Students should calculate or estimate their net income; e.g., allowances, and wages from part-time jobs.
C.6 To determine and calculate monthly fixed expenditures.	Determine fixed and variable expenses so that a budget can be planned. Students may have their own phones or car payments that they must make monthly. Have them investigate other expenses that they may have. Following a request for permission, students may wish to determine the monthly fixed expenditures of their families, if they do not have any of their own expenses.
C.7 To investigate the guidelines in developing a budget.	Net income varies from person to person so it is easier to compare the amounts budgeted with others by expressing all budgets in terms of percents. How do they arrive at these percents? What might affect these percents?
C.8 To plan a budget based on percentages allotted to various categories as suggested by financial institutions.	A budget can be planned by having students research the cost of an apartment, utilities, food, and transportation. Based on this budget, what monthly salary would be required to meet these expenses? Once the suggested guidelines had been researched, students can calculate the percentage of their monthly income allotted to each category. They can compare these results to the stated percentages from the Credit Union and then make any adjustments that they see fit.

Examples/Activities	Adaptations
Encourage students to find examples of graphs in newspapers and magazines that illustrate budgets; e.g., budgets for companies and organizations, and local, city, and provincial budgets. They can prepare a report on the information discussing the possible importance of the budget. (COM)	Students may be instructed to deal with budgets in various ways; as in budgeting for a 3-week holiday to a destination of their choice, for an assumed identity (e.g. the Prime Minister), or for some local situation (e.g. fishing lodge).
	The expenses can be done using the examples the students have developed above. Students should be able to express any or all costs/expenses as a percent of the entire budget.
Have students research guidelines in developing a budget. The Family Foundation and various financial institutions suggest percentages of a person's net income that should be allocated to various expenses. For example: savings: 10% to 20% Students may research further by visiting financial institutions for information on budgeting their money. For example: Credit Union. (IL)	
Students could work in groups to plan a budget for a shared-accommodation situation (or a class trip). This will involve all the information learned up to this point. They should be as realistic as possible in their estimations. Encourage students to research the costs of renting an apartment by using the newspaper. They can also investigate the cost of food, clothing, transportation, and entertainment in the same way. The use of spreadsheets on the computer could be utilized in this activity. (PSVS)	Budgeting, or planning, may also be adapted to focus on policy perspectives. As an example, northern students might have to "budget" the amount of equipment and supplies to take on a three-week `trapline' expedition.

Concept C: Consumer Mathematics

Objectives	Instructional Notes
C.9 To calculate the portion of total income spent on each category using percents.	This will involve a good understanding of percent. It will also show the relationship of each category to the whole.
C.10 To draw graphs (including circle graphs) of budget figures using appropriate software. (COM)	It is helpful for students to see a visual representation of their budgets. It will make them aware of how the categories compare to each other and as a part of a whole. Use of the calculator will come in handy when calculating the measure of the central angle as compared to the expenditure.
C.11 To calculate the actual amount of money to be spent on each category using the predetermined percentages.	This will allow students to compare their actual expenditures to the budgeted amounts. They will then be able to determine if they had over or under budgeted for each category. This objective requires a good understanding of how to work with percentages.
C.12 To adjust a budget to changes in expenses.	Budgets must be flexible in order to take into account unforeseen circumstances; e.g., car repairs and dental work. What categories could be changed to cover these unexpected expenses? This will require a great deal of thought and decision making on behalf of the students. (CCT)

Examples/Activities

In 1990-91 the Saskatchewan Health Care Budget (in millions) was spent as follows:

	$ (M)	%	Cum %
Doctors, etc.	262	17.4	17.4
Nursing Homes	221	14.7	32.1
Prescriptions	80	5.3	37.4
Other	286	19.0	56.4
Hospitals	658	43.6	100.0
	1 507	100.0	

Complete the % column for each expenditure and then calculate the cumulative totals. The percent of 1 circle can even be used to complete the circle graph. Draw lines at 0%, 17.4%, 32.1%, 37.4%, and 56.4% to complete the graph. Better students can multiply these percentage values by 360° to determine the angles. Using cumulative percentages (or angles) may be easier than calculating each individual central angle and adding it to the previous ending point.

Have the students work through an activity where an unforeseen expense arises. They will have to adjust their budgets accordingly. What categories could be adjusted to allow for this expense? What items in these categories are not crucial and could be left out for this month?

Adaptations

Make a circular protractor calculated in percents:
0% ---> 0°
50% ---> 180°
100% ---> 360° etc.
Calibrate in 5% patches.

A percent of a circle protractor can be useful in helping slower students catch on to drawing pie charts as they only need to calculate percents. They do not need to calculate the central angle in degrees.

Concept D: Lines and Line Segments

Foundational Objective

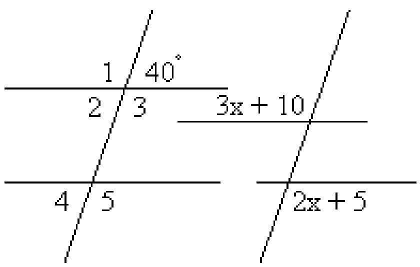
! To develop an informal understanding of the relationships between lines. (10 04 01). Supported by the following learning objectives.

Objectives	Instructional Notes
D.1 To define line segment, ray, line, bisector, median, perpendicular line, perpendicular bisector, transversal, alternate interior angles, corresponding angles, same-side interior angles.	
D.2 To identify and calculate the measures of corresponding angles, alternate interior angles, and same-side interior angles formed by parallel lines.	The physical world around us is full of examples of parallel lines. In pairs, have students describe and discuss real-world examples of parallel lines. They may also draw or take pictures of their examples. What occupations deal with parallel lines on a regular basis? Students may research or conduct interviews of various occupations. Why is it important to determine the measure of the angles in a pair of parallel lines? Encourage students to predict the relationship of the angles formed by parallel lines. They may test their hypotheses by using a protractor. (CCT) Use the relationships discovered to calculate the measures of the angles formed by parallel lines.
D.3 To solve problems involving angles formed by parallel lines.	Word problems involving parallel lines may be located in various resource texts. Students could work in pairs or in small groups to solve these problems. Solutions could be discussed by the entire class.
D.4, D.5, D.6, D.7 Informally and formally construct: a) congruent segments; b) the perpendicular bisector of a line segment; c) a line perpendicular to a given line from a point not on the line; d) a line perpendicular to a given line from a point on the line; and, e) a line parallel to a given line through a point not on the line.	Introduce constructions by using paper folding and/or mira. As time permits, follow up by using a compass and straight edge. Also, discuss the historical significance of the classical mathematical instruments. Allow students to explore and discover the methods to construct these segments and lines. Draw two lines perpendicular to the same line. What do you notice about them? How can you use this fact to draw a line through P parallel to AB? .P .B A.

Examples/Activities

Adaptations

Girders in a building, lines in a parkade, streets, steps in a stairwell, flight paths of the Snowbirds.



Solve the following problems:

1. An underground sewer goes under two parallel streets a block apart. The city engineer measures the alternate exterior angles on the same side of the sewer line from the manholes to the curbs, and finds a difference of 10° in their size. What are the measures of the angles formed by the curbs, manholes and sewer lines?

Students can create a Treasure Hunt using these constructions. They begin at a starting point and then progress from one point to another by constructing one of the lines that is described. Students must use their creativity and imagination to describe a situation that would motivate and interest other students in the class. The Treasure Hunts can then be exchanged and completed by classmates for practice and reinforcement. (CCT)

Include constructions involving special segments and circles; e.g., constructing a tangent to a circle at a given point on the circle, dividing a segment into a specified number of congruent segments, and circumscribing a circle about a triangle.

Concept E: Angles and Polygons

Foundational Objectives

- ! To identify and apply common properties of triangles, special quadrilaterals, and n-gons. (10 05 01). Supported by learning objectives 1, 2, 3, 4, 5, 8, 9, and 10.
- ! To apply the special quadrilaterals to real-world situations. (10 05 02). Supported by learning objectives 6 and 7.
- ! To develop an understanding of Pythagoras' Theorem, the primary trigonometric ratios, and their applications. (10 05 03). Supported by learning objectives 11 to 17.

Objectives	Instructional Notes
E.1 To define and illustrate by drawing the following: acute angle, right angle, obtuse angle, straight angle, reflex angle, complementary angles, supplementary angles, adjacent angles, vertically opposite angles, congruent angles, central angles of a regular polygon.	These terms are a review of the middle level courses. Review and extend where necessary. Have students estimate the measure of angles; especially those of 30°, 45°, 60°, 90°, 180°, and 360°.
E.2 To solve word problems involving the angles stated above.	Translation of word problems may be used to introduce the angles stated above. Students can also create their own problems that could be used for practice. Encourage students to use a variety of problem solving strategies; e.g., guess and checking or representing the problem with an appropriate diagram. (CCT)
E.3.(a) To define and illustrate the following polygons: convex, non-convex, regular, triangle, quadrilateral, parallelogram, rectangle, rhombus, square, trapezoid, isosceles trapezoid.	Students have classified and named polygons in the middle level (triangle, quadrilaterals, pentagons, hexagons, heptagons, octagons, nonagons, and/or decagons). A closer look at their properties involving sides, diagonals, and angles should be investigated at this level.
E.3.(b) To define and illustrate the following triangles: scalene, isosceles, equilateral, acute, right, obtuse.	This material is a review of grade 8 and 9. Review where necessary. Investigate whether a triangle could be described in more than one way; e.g., isosceles right triangle.

Examples/Activities

Have students research various careers in the library that require a knowledge of angles; e.g., surveyors, pilots, meteorologists, architects, cartographers, and carpenters. Reports may include oral presentations, charts, posters, essays, and skits. (COM)

The sum of the measures of the angles of a triangle is 180°. The first angle is 6 times as large as the second. The third angle is equal to the difference of the other two. What is the measure of the smallest angle? What are the measures of the other two angles?

$x + 6x + (6x - x) = 180$

In pairs, students classify several cut out triangles of varying sizes and shapes. Have them discuss the attributes they used to classify the triangles. Students should also evaluate the classification schemes.

Adaptations

Students could plan a panel discussion of three or four people from occupations that require a knowledge of angles. One student could be the moderator while the other students could prepare and ask relevant questions. (COM)

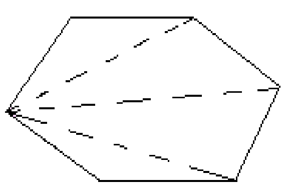
Concept E: Angles and Polygons

Objectives	Instructional Notes
E.4 To classify quadrilaterals as trapezoids, isosceles trapezoids, parallelogram, rectangles, rhombuses, and squares.	An inquiry approach may be used to classify quadrilaterals. Searching for commonalities and recognizing distinguishing characteristics are important problem solving strategies. Supply students with quadrilaterals of varying sizes and shapes so they may classify them according to an attribute of their choice. The students can then describe and discuss the reasons for their classifications. Encourage the students to evaluate the classifications and to determine which ones are most appropriate. They should also be able to recognize that certain quadrilaterals share common properties. Encourage students to develop a hierarchical classification. (CCT)
E.5 To informally construct parallelograms, rectangles, rhombuses, and squares.	Mira and paper folding may be used. If a more formal approach is taken, then a discussion of constructing angles and parallel lines with a compass and straight edge would be necessary.
E.6 To state and apply the properties of parallelograms: a) opposite sides are parallel b) opposite sides are congruent c) opposite angles are congruent d) the diagonals bisect each other.	Students may already have determined these properties from the preceding objectives and activities. They may reinforce their beliefs by using a mira.
E.7 To determine the sum of the measures of the interior and exterior angles of a convex polygon with n sides.	Allow students to discover this for themselves. An inductive approach may be used to conclude that the sum of the measures of the interior angles of a convex polygon with n sides is $(n-2)180^\circ$ and that the sum of the measures of the exterior angles is 360°. Students may work with a partner or in small groups to arrive at their conclusions. Allow them to share their ideas with the entire class.

Examples/Activities

Devise a game that students can play that involves recognizing the properties of quadrilaterals and other polygons. Create a game card of various geometric shapes consisting of quadrilaterals, triangles, and other polygons. Students work in groups of three: two players and a referee. Each player draws a geometric figure from an envelope containing the cut-up game card pieces. The objective is to guess the opponent's figure correctly before he/she guesses yours by asking a series of alternating questions regarding the properties of the figure. Each question must have a yes or no answer and include only one property. The referee determines if the questions and answers are valid. (CCT)

Use triangular pieces to form as many polygons as possible. Use the pieces to determine the sum of the measures of the angles of the convex polygons. Care must be taken to ensure that the students recognize that the sum of the angles of all the triangles is actually the sum of the interior angles of the polygon. Some students will deny this; e.g.,

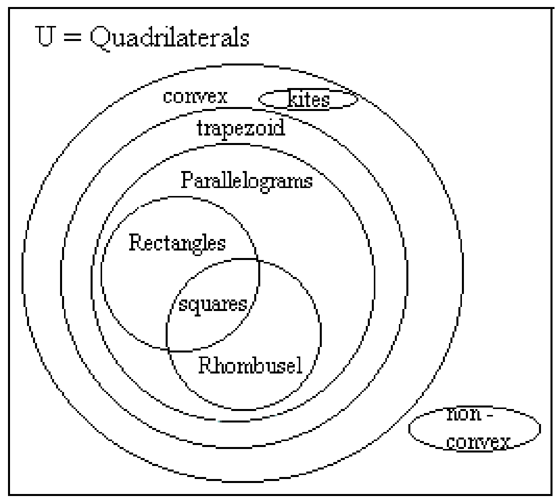


Adaptations

Investigate line or rotational symmetry.

Investigate the properties of the kite.

A classification that might be introduced from quadrilaterals is the following Venn diagram. It might also be used to help classify and sort out properties of parallelograms



Concept E: Angles and Polygons

Objectives	Instructional Notes
E.8 To determine the measure of a central angle in a regular n-gon.	An inductive approach may be used to carry out objectives E.8, E.9, and E.10. Students will need a protractor and examples of several regular polygons. Students should realize that in a regular polygon the measure of an exterior angle is equal to the measure of the central angle.
E.9 To determine the measures of the interior and exterior angles of regular n-gons.	
E.10 To determine the number of diagonals in a polygon of n sides.	
E.11 To calculate to two decimal places the length of the missing side of a right triangle using the Pythagorean Theorem.	Students have used the Pythagorean Theorem in calculating the length of a missing side of a right triangle in grades 8 and 9. However, the calculations were restricted to the roots of perfect squares. Students can develop an intuitive understanding of square roots by estimating the value before using the calculator. Students should be aware what each variable represents and that other variables may be used in the formula.
E.12 To solve word problems using the Pythagorean Theorem.	Use realistic word problems to which students may relate. The students can also identify situations where the Pythagorean Theorem may be used. In groups, the students can create their own word problems. The groups may then exchange problems for the required practice and reinforcement. (COM)

Examples/Activities	Adaptations

# of sides	# of diagonals	# of triangles	sum of interior angles	m. of int. of regular polygon	m. of ext. of regular polygon	m. of centre angle
3 4 5 6 7 8 9 10 • • • n						

m. = measurement

$c^2 = a^2 + b^2$
$d^2 = e^2 + f^2$

Solve each of the following:

1. How far up a wall will a 10 m ladder reach if the base of the ladder is 2 m from the wall?

2. Jason rides his motorcycle at a speed of 80 km/h due south and Jan drives her car at a speed of 100km/h due east. How far apart will they be after 2 hours?

3. Chad uses a guy wire to support a young tree. He attaches it to a point 2 m up the tree trunk. The wire is 3 m long. How far away from the trunk will the wire reach?

Calculate the lengths of the legs of a right triangle when given the ratio of the legs and the length of the hypotenuse; e.g., The dimensions of the viewable area of a television are in the ratio 4:3. If the length of the viewable diagonal is 70 cm, what are the dimensions of the viewable area of the television?

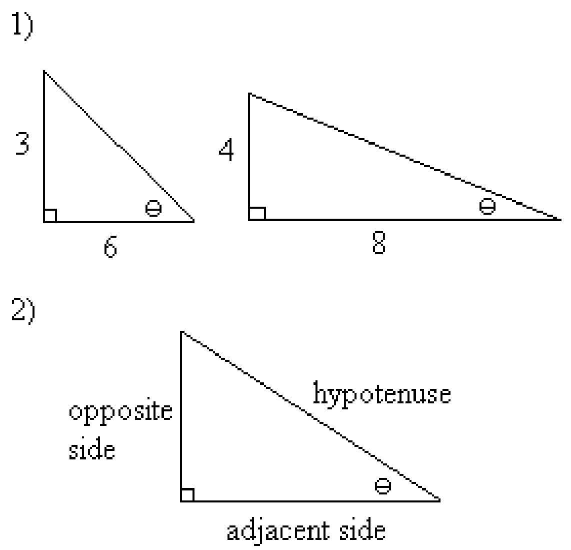
Concept E: Angles and Polygons

Objectives	Instructional Notes
E.13 To determine if a triangle is a right triangle by using the converse of the Pythagorean Theorem.	Students should investigate various triangles with given lengths to determine if the Pythagorean Theorem works for all triangles. How can you determine if a triangle is a right triangle when given only the lengths of the sides?
E.14 To determine the value of the three primary trigonometric ratios by using a calculator.	Review and extend similarity from the middle level. Use an informal approach to similarity in the discussion of the tangent, sine, and cosine ratios. These ratios may be introduced by using various sized similar right triangles having the same acute angles. Students can observe and discover how the ratios of the sides compare to the acute angle. Emphasize that the values for the trigonometric ratios depend on the size of the angle and not on the size of the triangle. Students should be encouraged to use words like adjacent, opposite, and hypotenuse in their discussion of the ratios. They may also experience difficulty in identifying the opposite and adjacent sides with respect to a particular acute angle. The students can cut out a right triangle from a piece of paper and label an angle. The adjacent side, opposite side, and hypotenuse may then be labelled and referred to when needed. Introduce the use of the calculator at this point. (COM)
E.15 To determine the measure of an angle given the value of one trigonometric ratio of the angle using a calculator.	Students should be able to determine the measure of an angle when given the value of the trigonometric ratio. Explain how to find the measure of an angle from a trigonometric table. Also explain how to use the inverse key on the calculator. Advise students to round the angles off to the nearest degree.
E.16 To calculate the measure of an angle or the length of a side of a right triangle using the tangent, sine, and cosine ratios.	Attention should be drawn to the given information to determine which ratio should be used to calculate the missing part of a right triangle. Point out that the Pythagorean Theorem may also be used depending on the information given.

Examples/Activities	Adaptations

Determine if the corner of a room is a right angle. Devise a plan and carry it out. Reflect back on the reasonableness of the answer.

Calculate the diagonal of a rectangular solid (parallelepiped) given the three sides.



Have students draw any right triangle that has acute angles that measure 30° and 60°. Measure and label the angles and sides of the triangle. Display all of their triangles on the board. Have the students compare the various ratios of the sides of the triangles. What conclusions can be made from these results?

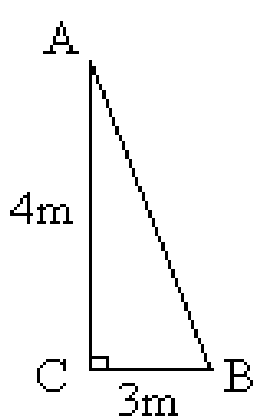
Explore the reciprocal trigonometric ratios.

Discuss how to construct an equilateral triangle with a compass (or mira, paper folding). How is this helpful in making a 30°-60°-90° triangle?

Repeat the activity with an isosceles right triangle.

If tan A = 0.4663 then find the m< A.

If one has a right triangle, what are some possible values for the two acute angles? What do you notice about these pairs? (complementary)



1) Calculate m<B
2) Find AB

Concept E: Angles and Polygons

Objectives	Instructional Notes
E.17 To solve problems that involve trigonometric ratios, using a calculator.	Use problems related to such occupations as navigators, surveyors, architects, and engineers. Include examples of angles of elevation and angles of depression. Involve students in a discussion of why the angle of elevation is congruent to the angle of depression in a diagram. Encourage students to use a variety of problem-solving approaches. Drawing and labelling a diagram is crucial in determining the correct trigonometric ratio.

Examples/Activities

Solve each of the following:

1) The angle of elevation from a boat to the top of an iceberg is 75°. If the boat is 200 m from the base of the iceberg how high is the iceberg?

2) A forest ranger in a tower 28 m high sights a fire at an angle of depression of 6°. How far is the fire from the tower?

3) Brent has to look up 70° to see the top of a tree 4 meters away. If his eyes are 1.8 m above the ground, how tall is the tree?

Adaptations

Have student check their answers for reasonableness. Working in pairs or in small groups will help reinforce this aspect. (PSVS)

Teacher Notes

Concept F: Review of Algebraic Skills

Note: The objectives that follow are a review of grade 9. A review is necessary before Mathematics 20. Review and expand where necessary. Diagnostic evaluation may be appropriate to gather information on students and to assist in determining the most effective method of instruction. Based on these results, instruction may include: teaching to the whole class with the same assignment or with differentiated assignments; teaching students in groups that work on different lessons; and, using continuous progress where students work at their own rate. Certain students may only have to review areas of weakness.

Foundational Objective

! To make the transition from arithmetic skills to algebraic skills. (10 06 01). Supported by the following learning objectives.

Objectives	Instructional Notes
F.1.(c) To add and subtract rational numbers.	This includes integers, fractions, and decimals. Be sure to include negative subtrahends when subtracting integers. Where are rational numbers used in everyday life?
F.1.(d) To multiply and divide rational numbers.	Encourage students to use estimation and mental computation as much as possible. Approximations should be used to check the reasonableness of calculator answers. Terms like mixed number, numerator, denominator, mixed fraction, improper fraction, lowest common denominator, reciprocal, and divisor should be reviewed. (COM)
F.1.(e) To use the order of operations in evaluating rational arithmetic expressions.	Review the order of operations. Students should also explore the order of operations with their calculators. Calculators vary in their ability to perform the order of operations.

Examples/Activities

1) 2 + -15 + 8 + -23
2) -56 - (-47)
3) -2/3 + -5/8
4) 2 2/5 - (-1 1/2)
5) 2.065 - 14.45

Have students explore the uses of rational numbers by investigating various occupations. They could conduct interviews with people of various occupations by asking them how they use numbers or mathematics in general in their line of work. The information could be organized and displayed on the bulletin board for all to view. (COM)

1) (-12)(2)(-3)
2) (-1 2/3)(6/25)
3) (12.01)(3.4)
4) 240 ÷ (-18)
5) 7/8 ÷ (-5 1/3)
6) -9.5 ÷ 0.32

Math baseball or some other interesting and motivating game can be used to review all the operations dealing with rational numbers.

1) $2 + [4 - (7 - 10)^2] - 5^2$

2) $-\frac{2}{3}\left[\frac{3}{2} \div \left(1\frac{1}{2} - 1\right)\right] + \frac{2}{3}$

Adaptations

Encourage students to lump fractions into 3 groups:

! about 1,
! about 1/2, and
! about 0.

Use these values to estimate the reasonableness of answers.

Concept F: Review of Algebraic Skills

Objectives	Instructional Notes
F.2.(a) To evaluate a positive power of a numerical base.	Review factor, base, exponent, and power. Encourage the use of the calculator.
F.2.(b) To evaluate multiplication and division of positive powers of the same numerical base.	Students should be able to express each expression as a power with a single base. A calculator can then be used to evaluate the expression.
F.2.(c) To perform the product and quotient exponent properties with variable bases.	Be sure students have a good understanding of multiplying and dividing positive powers of the same numerical base before working with variable bases.
F.2.(d) To write numbers in scientific notation and vice versa.	Review why and where scientific notation is used. Students should be able to determine the relationship between the movement of the decimal and the sign on the exponent. Explore the use of scientific notation on the calculator. (NUM)
F.2.(e) To perform multiplication and division of numbers expressed in scientific notation.	Negative exponents have not been introduced at this point so exponents should be restricted to the natural numbers.

Examples/Activities

4^5

$(2^{13})(2^7)$

1) $(x^6)(x^5)$
2) $\frac{m^{12}}{m^8}$

Note: $\frac{10^7}{10^4} = 10^3$, , not 1^3

1) $2456 = 2.456 \times 10^3$
2) $3.49 \times 10^5 = 349\ 000$

$(2.4 \times 10^3)(3.0 \times 10^2)$
$= (2.4 \times 3.0)(10^3 \times 10^2)$
$= 7.2 \times 10^5$

Note: $(10^4)(10^5) = 10^9$, not 100^9

Adaptations

7^{3274} ends with what digit?

Teacher Notes

Concept F: Review of Algebraic Skills

Objectives	Instructional Notes
F.3. (a) To add and subtract polynomials.	Review the definitions for variable, numerical coefficient, literal coefficient, term, like terms, monomial, binomial, trinomial, and polynomial. Also review expressing polynomials in decreasing or increasing order of powers for one of the variables. The use of manipulative blocks or algebra tiles may help students to develop a better cognitive understanding of polynomials. They may be used to perform all the operations with polynomials. Remind students that polynomials represent numbers. Attention should be given to subtracting terms in parenthesis. The rule of subtraction may be used.
F.3. To multiply: b) a monomial by a monomial c) a polynomial by a monomial d) a binomial by a binomial	Relate multiplication of polynomials to calculating the area of a rectangle with varying dimensions. Manipulative blocks or algebra tiles may also help to develop understanding by students who have not developed their abstract thinking. This will naturally lead into a review of the distributive property. An alternate prescription to expanding binomials is to use mnemonics; e.g., FOIL. Students may be at different levels of understanding and they should be encouraged to use the method that makes most sense to them. Encourage the use of mental calculations for simple products. Multiplying two binomials is an excellent way of bridging arithmetic to algebra. (NUM
F.3. To divide: e) a monomial divisor f) a polynomial by a monomial	Review the quotient property for the laws of exponents. Emphasize that each term in the numerator must be divided by the denominator. The use of common factors may be used to illustrate this process. The long division algorithm for whole numbers may also be used.

Examples/Activities **Adaptations**

1) $2x + 5x^2 + 6x + 7x^2$
2) $13xy + 2x^2 + 3xy + y^2 + 5x^2 + 4y^2$
3) $-(9y^2 - 7y + 1)$
4) $(8x - 3) - (5x - 2)$

1) $(x + 2)(x + 7)$

	x	+ 7
x	x^2	7x
+ 2	2x	14

2) $(x + 2)\ (2x + 7) = 2x^2 + 11x + 14$. Since this is true for all values of x, we can check by letting x = 10. (Note that this is really multiplying 12 x 27.)

$$(10 + 2)[2(10) + 7] = 2(10)^2 + 11(10) + 14$$
$$(12)(27) = 2(100) + 110 + 14$$
$$324 = 200 + 110 + 14$$
$$324 = 324$$

This also provides practice in substituting values in an expression and helps keep arithmetic skills sharp.

$$\frac{6x^3 + 9x^2 + 15x}{3x}$$

Check by letting x = 10. (Same as dividing 7050 by 30.)

Mathematics 20 Curriculum

Foundational Objectives - Mathematics 20

The Foundational Objectives describe the most important understandings and abilities which should be developed over the course of a unit or a year. They provide guidance to teachers in unit and yearly planning and should be within the range of abilities of the majority of students.

The Foundational Objectives form the basis for curriculum evaluation.

The following Foundational Objectives outline the important understandings to be developed in Mathematics 20.

- To identify an irrational number and to demonstrate the ability to add, subtract, multiply, and divide square root radicals. (10 01 01)
- To demonstrate an understanding of credit and to employ the appropriate mathematics in determining the cost to the consumer of various types of credit. (10 02 01) (NUM)
- To display an awareness of the kinds of taxes encountered by the consumer and to demonstrate the ability to calculate these taxes using the appropriate mathematics. (10 02 02)
- To demonstrate the ability to factor polynomial expressions, including trinomials of the type ax^2+bx+c . (10 03 01)
- To demonstrate the ability to simplify expressions correctly that contain positive and negative integral exponents. (10 03 02)
- To demonstrate the ability to add, subtract, multiply, and divide rational expressions with monomial denominators. (10 03 03)
- To be aware that the graph of an equation in two variables, where only one variable is of degree two, is a parabola. (10 04 01)
- To draw graphs of equations representing parabolas. (10 04 02)
- To demonstrate an ability to interpret an equation of the form $y=a(x-p)^2+q$, as to the effect the values of a, p, and q have on the equation. (10 04 03)
- To demonstrate the ability to solve quadratic equations by factoring, and by taking the square root of both sides of a quadratic equation. (10 05 01)
- To demonstrate the ability to solve equations containing one radical. (10 05 02)
- To appreciate the role of probability in understanding everyday situations. (10 06 01) (NUM)
- To develop the ability to identify pairs of congruent triangles, and to employ the congruence postulates SSS, SAS, ASA, AAS, or HL in guided proofs showing such congruences. (10 07 01)
- To demonstrate the ability to apply the concepts of similar polygons and scale factors to determine the surface area and/or volume of similar polygons or solids. (10 07 02)
- To provide a reasonable explanation for congruences of pairs of triangles, or for corresponding parts of congruent triangles. (10 07 03)
- To develop and display an understanding of certain relationships of the chords, tangents, and arcs of a circle. (10 08 01)

The development of the following Foundational Objectives for the Common Essential Learnings does not preclude the development of other C.E.L.s objectives.

The Common Essential Learnings (C.E.L.s) are coded throughout this document as follows:

CCT - Critical and Creative Thinking
COM - Communication
IL - Independent Learning
NUM - Numeracy
PSVS - Personal and Social Values and Skills
TL - Technological Literacy

- To enable students to understand and use the vocabulary, structures, and forms of expressions which characterize mathematics. (COM)

- To strengthen students' understanding within mathematics through applying knowledge of numbers and their relationships. (NUM)

- To strengthen students' knowledge and understanding of **how** to compute, measure, estimate, and interpret numerical data, **when** to apply these skills and techniques, and **why** these processes apply within mathematics. (NUM)

- To support students in treating themselves, others and the environment with respect. (PSVS)

- To develop students' abilities to access knowledge. (IL)

- To develop a contemporary view of technology. (TL)

- To promote both intuitive, imaginative thought and the ability to evaluate ideas, processes, experiences, and objects in meaningful mathematical contexts. (CCT)

Western Protocol - General Outcomes Grades 10 - 12

Saskatchewan Education, Training and Employment is part of a Western Protocol agreement to develop a common curriculum framework in several areas, including mathematics. The following General Outcomes are taken from the Western Protocol, *Common Curriculum Framework for Mathematics* (draft, March 1995) and are presented here for your awareness.

Numbers
Number Concepts

- Explain and illustrate the structure and the interrelationship of the sets of numbers within a number system.
- Describe and apply arrays in the formulation and solution of problems.
- Describe, apply, and transform arrays in the formulation and solution of problems.

Number Operations

- Apply the basic operations singly or in combination, (and their interrelationship) on any real number and illustrate the use of these operations in solving real-world problems, such as budgeting.
- Describe and apply operations on arrays to solve problems, using spreadsheet technology as required.
- Describe and apply operations and transformations on arrays to solve problems, using spreadsheet technology as required.

Patterns and Relations
Patterns

- Represent by models, naturally-occurring data, using linear and nonlinear functions.
- Represent by models, those situations involving natural growth and decay.
- Represent by models, naturally-occurring data that are cyclical in character.

Variables and Equations

- Represent and analyze situations that involve variables, expressions, equations, and inequations.
- Connect and analyze functions and their graphs to solve problems.
- Translate between algebraic and coordinate relationships and the corresponding geometric shapes to solve problems.

Relations and Functions

- Translate between the various representations of discrete and continuous functions.
- Link models involving natural growth and decay to the exponential function.
- Link models involving data of a cyclical character to trigonometric functions.

Shape and Space
Measurement

- Solve problems using modern measuring devices such as electronic tuning forks, calipers, micrometer, electronic tape measures, and instrument-computer interfaces.
- Use the special properties of right triangles, including the sine, cosine, and tangent ratios, to solve problems involving more than one right triangle.
- Design an object to satisfy constraints such as maximum or minimum.
- Investigate the combination of vector quantities.
- Apply the sine and cosine laws to solve real-life problems involving general triangles.

3-D Objects and 2-D Shapes

- Justify congruence and similarity using transformations and coordinate geometry.
- Derive the properties that relate chords, tangents, and arcs and angles in circular and spherical geometry.
- Make and justify conjectures regarding the properties of a set of related figures.

Transformations

- Apply coordinate geometry and pattern recognition to predict the effects of combinations of transformations on 1-D and 2-D objects.
- Represent transformations numerically.

Statistics and Probability
Data Analysis

- Describe, implement, and analyze sampling procedures, and draw appropriate inferences from the data collected.
- Apply curve-fitting and correlation techniques to analyze experimental results.
- Analyze discrete and continuous data distributions to produce descriptions and inferences about populations and samples.

Chance and Uncertainty

- Use expected gains and losses to make and analyze decisions based on a single event.
- Model the probability of a compound event using the probability of simple events, and use these models to solve problems involving uncertainties.
- Solve problems based on the counting of sets or the combining of simpler probabilities.

Concept A: Irrational Numbers

Foundational Objective

! To identify an irrational number and to demonstrate the ability to add, subtract, multiply, and divide square root radicals. (10 01 01). Supported by the following learning objectives.

Objectives	Instructional Notes
A.1 To define and illustrate, by means of examples, the term absolute value.	The definition of absolute value refers to distance. This should first be shown using number line examples. The abstract approach and notation can be introduced once students understand the concept.
A.2 To express square root radicals as mixed radicals in simplest form.	The concept of square root is to be introduced in the middle level math curriculum. A review of the squares of the more common numbers may help students recall this topic. Have students practise finding the largest square which is a factor of a given number. Establish the meaning of $\sqrt{\ }$, that being a positive or principal square root. Also, introduce $\pm\sqrt{\ }$, and $-\sqrt{\ }$. You might discuss $\sqrt{-}$, although it is not a real number.
A.3 To add, subtract, multiply, and divide square root radicals.	For addition and subtraction, it must be stressed that the radicands must be equal before the operation is attempted. Some practice in identifying such radicands can precede the actual operations of addition and subtraction. When introducing multiplication and division, stress that students must check their answers to see if they are able to simplify the radicand.

Examples/Activities	Adaptations

1. Illustrate the following on a number line (as a distance from zero), and state the results. a) $|4|$ b) $|7|$

2. What is the value of each of the following:

$a)\ |3| \quad b)\ -|6| \quad c)\ |4|+|-3| \quad d)\ |-4|+|8|$

$e)\ -|6| + |-9|$

1. $\sqrt{12}$ 2. $\pm\sqrt{18}$ 3. $-\sqrt{50}$

4. $\sqrt{-32}$ 5. $\sqrt{125}$ 6. $\sqrt{96}$

7. $\pm\sqrt{63}$ 8. $4\sqrt{12}$ 9. $5\sqrt{8}$

10. $10\sqrt{63}$

This section could be extended by including variables.
For example:

$\sqrt{32x^5}$ or $\sqrt{x^6y^7}$

A discussion on permissible and non-permissible values of the variables should also be held.

1. $\sqrt{27} + \sqrt{12} + \sqrt{48}$

2. $6\sqrt{5} - \sqrt{125}$

3. $2\sqrt{3} \cdot 6\sqrt{5}$

4. $6\sqrt{15}$

5. $4\sqrt{2} \cdot 3\sqrt{6}$

6. $\dfrac{\sqrt{27}}{\sqrt{12}}$ 7. $\sqrt{\dfrac{36}{6}}$

8. $\dfrac{4\sqrt{15}}{2\sqrt{3}}$

Extend by introducing variables throughout this series of operations.

Concept A: Irrational Numbers

Objectives	Instructional Notes
A.4 To rationalize monomial denominators.	Students should have some practice squaring irrational numbers to make them rational before incorporating this procedure into division. Because it is easier to divide an expression by an integer instead of an irrational number, mathematicians have used the convention of `rationalizing the denominator', in those cases where it does not divide evenly (ie. there is a remainder) into another irrational number in the numerator.

Examples/Activities

Practice:

1. $\sqrt{7} \cdot \underline{?} = 7$ 2. $\sqrt{3} \cdot \underline{?} = 3$

3. $\sqrt{6} \cdot \underline{?} = \_\_$ 4. $\sqrt{12} \cdot \underline{?} = \_\_$

(discuss acceptable answers)

1. $\frac{1}{\sqrt{3}}$ 2. $\frac{2}{\sqrt{5}}$

3. $\frac{6}{\sqrt{3}}$ 4. $\frac{-5}{\sqrt{2}}$

5. $\frac{-7}{2\sqrt{3}}$ 6. $\frac{4}{3\sqrt{8}}$

Adaptations

Extend by introducing variables into the procedure.

Teacher Notes

Concept B: Consumer Mathematics

Foundational Objectives

- To demonstrate an understanding of credit and to employ the appropriate mathematics in determining the cost to the consumer of various types of credit. (10 02 01). Supported by learning objectives 1 to 10.

- To display an awareness of the kinds of taxes encountered by the consumer and to demonstrate the ability to calculate these taxes using the appropriate mathematics. (10 02 02). Supported by learning objectives 11 to 14.

Objectives	Instructional Notes
B.1 To define credit and determine its appropriate use.	Students could be asked through brainstorming in small groups, to provide the names of some types of credit. A list could be formed from the answers of the entire class. Alternatively, a loans officer from a financial institution may be invited to present this material to the entire class. This might also be done by using student-conducted interviews. (COM)
B.2 To compare the advantages and disadvantages of various credit cards.	The students or the teacher should acquire credit card information, through credit card application forms, financial institutions, information booklets, consumer magazines, or actual credit card statements. (IL) Alternatively, a representative of a financial institution, versed in these areas, may be used as a classroom resource.
B.3 To calculate the monthly interest charges and service charges on an unpaid credit card balance.	Students may need some review of percent and interest calculations. A variety of methods should be encouraged, including use of calculators, computers, tables, and formulas.

Examples/Activities	Adaptations
Students could work in groups to discuss the situations in which each type of credit is most appropriate. Types of credit to discuss might include: 1. credit cards- Mastercard, Visa, gasoline, retail, etc., 2. instalment plans, 3. personal loans, 4. line of credit, 5. mortgages, 6. farm, 7. vehicle/equipment, 8. consumer loans, 9. investment loans, 10. demand notes.	Students might be asked to interview a loans officer of a financial institution to determine their position regarding the various types of credit. It might be possible to arrange a panel discussion of loans officers from several financial institutions. Students could also obtain information on various types of credit cards. Reports of their research could be shared with the entire class. If available, a bill collector might be asked to discuss credit abuses.
Have students compare the information available from various sources, to list advantages and disadvantages of each, and to state why they perceive each as an advantage or disadvantage. Each student should share his/her point of view with others, as discussion might be useful. (CCT) Some things to consider are: interest rates, universality, usage fees, credit limit, acceptability, and special discounts.	Students could be presented with several personal profiles that might be generated by the students themselves. A character sketch could be written for each profile, and a list of credit card choices for this person would be given, complete with reasons for this choice of credit cards. Alternatively, the teacher could make up case studies of credit card users that have overextended themselves. Discuss the mathematical, family, and social implications of their debts. There are some films available from Consumer and Corporate Affairs that deal with financing and credit.
Students might be provided with a blank credit card statement, showing yearly, monthly, and daily interest rates. Different balances could be used to determine interest charges. Service charges, if any, should also be included.	

Concept B: Consumer Mathematics

Objectives	Instructional Notes
B.4 To identify and compare an instalment charge account and a thirty-day account.	Define these terms for the class. Students should research the characteristics of these two accounts, and report their findings to the class. An instalment charge account is one in which a contract is agreed to, in which a monthly payment (instalment) is stipulated. The instalment includes interest and service charges. The instalments continue until the agreed total amount is paid. A thirty-day account is one in which credit is given for thirty days, and then must be paid. Any balance has interest charged. Payments vary month to month as credit is given. A guest speaker drawn from the business or financial community may be asked to address this topic.
B.5 To identify the characteristics of a personal loan.	Information should be gathered from financial institutions, financial, or consumer magazines. This could be done by students as a research assignment, or by the teacher. This information could then be presented to the class. Alternatively, a guest speaker from a financial institution could be used as a resource.
B.6 To compare the cost of a consumer loan from various institutions.	Most financial institutions have pamphlets readily available on costs of consumer loans. The teacher or students should acquire several different ones to use as comparison.
B.7 To calculate the monthly payments for a loan using formulas, tables, calculators, and computers.	A representative of a financial institution might be invited to discuss the methods he/she would use to calculate monthly payments for a loan.

Examples/Activities	Adaptations
Students could be asked to generate a list of characteristics of a personal loan after searching through the gathered information, or after listening to a resource speaker. This could be done in small groups, with reports being given by each group, and a summary conclusion done by the entire class. Reports could include visual displays, such as charts and diagrams. (COM)	
Students could be given the information, and asked to calculate the cost of a loan of x dollars over differing periods of time. Note that some interest rates change with the amount of the loan, so that various types of dollar figures should be used. Students should be encouraged to utilize several methods of calculation.	Because cost is not the only consideration when incurring a consumer loan, students could be asked to reason out why there might be differences, and under what circumstances a higher cost might be a suitable alternative. Students could be asked to determine the cost of an item they wish to purchase. They would be expected to find this information through newspapers, magazines, or interviews, and then carry out calculations based on their research. This could be used for objectives 6, 7, and 8. (IL)
Students could be given a sample question and different groups assigned different methods of solution. A comparison of the answers obtained by the various groups may provide some excitement.	

Concept B: Consumer Mathematics

Objectives	Instructional Notes
B.8 (a) To determine the total cost of a personal loan.	Students could generate the definition of total cost through a class discussion, or it could be provided directly by the teacher.
(b) To determine the percent of the total amount repaid (or borrowed) which is devoted to interest.	Students could use the same examples they completed in the previous section.
B.9 To determine the importance of a credit rating.	A loans officer from a financial institution may be invited to speak to this topic. Alternatively, students could be instructed to research this aspect, either through interviews, or by reading, and report their findings to the class. (COM)
B.10 To describe how to establish a good credit rating.	This could be combined with the previous section. The same methods could be employed.
B.11 To calculate mill rates and property taxes.	Definitions of new terms should be provided. Formulas should be given, and variables described. Several methods of calculation should be encouraged. (NUM)
B.12 To calculate discounts or penalties on taxes due, depending on when they are paid.	Introduction of new terms should precede the lesson itself.

Examples/Activities	Adaptations
1. What is the total cost of a loan of $5 000 borrowed for two years at a rate of interest of 11% per annum? 2. What is the total cost of a personal loan of $15 000 borrowed for four years at a rate of interest of 12% per annum?	This topic could be extended by having students include service charges such as registering and deregistering a loan, in the total cost of the loan. Students might also consider 'lost interest' on their money, as another hidden cost of loans.
1. If James paid back a total of $6 250 on a loan of $5 600 for one year, what percent of the repayment was devoted to interest? 2. If Ila paid back $675 on a loan of $625 in six months, what is the percent devoted to interest? What was the rate of interest?	
A discussion could be held on this topic. A list of characteristics that comprise a credit rating might be generated. The students could discuss the advantages of having a good credit rating. The discussion should also address the issue of whether certain groups of people have more difficulty in establishing a credit rating. ie. women, racial minorities, aboriginals. (PSVS)	This topic could be extended by having students speak to municipal or corporate officials, to determine the importance of a credit rating for a corporation or municipality or province.
Several fictitious personal profiles could be distributed for reading, or for role-playing. Students could then discuss whether each case would receive a good credit rating.	Students could explore some of the methods used by corporations, municipalities, or provinces to establish good credit ratings.
1. If Jon's property is assessed at $9 500, and the mill rate is set at 63, what taxes should Jon expect to pay on the property.	Students could be asked to interview municipal officials to determine what factors are taken into account when the mill rate is set, and how property is assessed.
1. If Graeme receives a tax assessment notice which states his taxes are $2 200 payable by June 30, what would he pay if he received a 3% discount for paying before April 30? What would he pay if he received a 1.5% discount if paid before May 30? What would he pay if a 2% penalty was levied for taxes not paid by July 30?	Students could be asked to interview municipal officials to determine what percentage of property owners received discounts, and what percentage paid penalties. If there are several categories in each case, these may be represented on a circle graph, and presented to the class. (NUM)

Concept B: Consumer Mathematics

Objectives	Instructional Notes
B.13 To determine and calculate permissible deductions from total income.	Income tax guides should be obtained for the students before beginning this topic. Students should be given some time in advance of the lesson to read through the guide. Several case studies could be prepared for class distribution.
B.14 To calculate the tax payable on taxable income.	Students should be familiarized with the tax tables, and how they are read. (NUM) They should be reminded that provincial tax, and any surtaxes on higher income earners, must be added to the federal tax to determine the total tax. Calculator usage should be encouraged.

Examples/Activities

Students can work in small groups, and list deductions from the guide that apply to the individual case studies. When they have listed the deductions, they may use the income tax form and guide to calculate the total value of the deductions.

1. If Amit's taxable income is $23 000, how much is Amit's total income tax bill?

2. If Kendra's taxable income is $43 575, what is her total income tax bill?

Adaptations

The Federal Department of Taxation has videos, workbooks, and other materials available to teachers. There are also computer programs for doing income tax.

It is important for all students to complete the tax section regardless of aboriginal status. All off-reserve employers are required to deduct taxes; only on-reserve employers of treaty Indians are not required to deduct income taxes. Most aboriginal students will probably at some time work off reserve and will be required to submit a tax return, and by doing so they may be entitled to special tax credits. Some treaty Indians receive their income tax back in the form of a refund. For more information refer to Native Studies 10, Student Guide, "Political Life" or the Teacher Guide, "Political Life: The Indian Act sections 86-89" or section 91 of the *British North America Act*.

Taxation that applies to persons of Aboriginal ancestry is an area that is in a state of flux.

Concept C: Polynomials and Rational Expressions

Foundational Objectives

- ! To demonstrate the ability to factor polynomial expressions, including trinomials of the type $ax^2 + bx + c$. (10 03 01). Supported by learning objectives 1 and 6.

- ! To demonstrate the ability to simplify expressions correctly that contain positive and negative integral exponents. (10 03 02). Supported by learning objectives 3 and 4.

- ! To demonstrate the ability to add, subtract, multiply, and divide rational expressions with monomial denominators. (10 03 03). Supported by learning objectives 2, 5, 7, and 8.

Objectives

C.1
To factor polynomials of the following types: common factor, grouping, difference of squares, trinomial squares, trinomials where $a = 1$, $a \neq 1$, and combinations of all preceding types.

Instructional Notes

A brief review of polynomial multiplication may be useful. Algebraic tiles (a manipulative), or paper-folding can be utilized to introduce the concept of factoring.

Students can be instructed to check each others' answers by multiplying the factors to see if the product is equal to the original question.

Trinomial factoring can be done using several different, methods, such as `trial and error', `decomposition', or `division'. Some calculators and computer programs can also be used for factoring. Students should be introduced to a selection of factoring methods.

C.2
To divide a polynomial by a binomial, by factoring, and long division.

Algebraic tiles or paper-folding may be utilized in this section.

Division by factoring can be presented as the inverse operation of multiplication.

Examples/Activities

Factor:

1. $4x+12$
2. $3y^2+5y$
3. $4y+12+xy+3x$
4. $6x^2-10x+9x-15$
5. x^2-9
6. $49r^2-16$
7. $(36)^2-(24)^2$
8. x^2+6x+9
9. $4t^2-28t+49$
10. $2x^2+5x+2$
11. $4x^2-3x-1$
12. $2x^2+6x-20$
13. $a^2-(x^2+4x+4)$

1. Divide by factoring:

   $a)\ (x^2-3x-10)\div(x-5)$
   $b)\ (3x^2-10x+3)\div(3x-1)$

2. $(x+3)\overline{)x^2+7x+12}$

3. $(a-2)\overline{)a^2-6a+7}$

4. $\dfrac{x^2-8x-15}{x-4}$

Adaptations

Students could be asked to supply missing values in a set of statements; For example:

1. $4(\_\_\_\_+3)=6x+12$
2. $x^2-8x+\_\_\_\_=(x-4)^2$
3. $25x^2-9y^2=(5x-3y)(\_\_\_\_)$
4. $x^2-7x+10=(\_\_\_\_)(x-2)$

Word problems can also be associated with factoring. The area of a rectangle and one dimension can be given, and the students directed to determine the missing dimension.

Other types of word problems may be taken from fields such as business or commerce.

Note that arithmetic substitution can be a good check of the accuracy of factoring. It also provides immediate reinforcement, practice in substituting in and evaluating expressions, and mental computations.

Modify for students who are experiencing difficulty by keeping the coefficient of the divisor equal to 1.

Concept C: Polynomials and Rational Expressions

Objectives	Instructional Notes

A review of the division algorithm, using numbers, may be useful; i.e., the process may be reviewed with an example of the type $23\overline{)715}$, $(20 + 3)\overline{)700 + 10 + 5}$. Students can check each others' answers by being instructed to multiply the divisor by the quotient, and adding the remainder. (They might be expected to develop this `check' on their own.)
A brief review of 'the meaning of an exponent' should be given.

C.3
To evaluate powers with positive and negative exponents.

Students could be directed to find answers to numerical questions, such as 2^5, 4^3, etc. They might use calculators to determine answers to questions such as $(13)^3$, or $(23)^2$.

Students might be asked to complete a series of questions in chart form (as given below), to determine the value of an expression with an exponent of zero, and to extend this pattern to negative exponents.

$2^4 =$ __	$3^4 =$ __	$4^4 =$ __	$5^4 =$ __
$2^3 =$ __	$3^3 =$ __	$4^3 =$ __	$5^3 =$ __
$2^2 =$ __	$3^3 =$ __	$4^2 =$ __	$5^2 =$ __
$2^1 =$ __	$3^1 =$ __	$4^1 =$ __	$5^1 =$ __
$2^0 =$ __	$3^0 =$ __	$4^0 =$ __	$5^0 =$ __
$2^{-1} =$ __	$3^{-1} =$ __	$4^{-1} =$ __	$5^{-1} =$ __
$2^{-2} =$ __	$3^{-2} =$ __	$4^{-2} =$ __	$5^{-2} =$ __

Students could be asked to share their answers and observations in small group discussions, and determine if there are any patterns or generalizations they might make. It is important to ensure that students treat each other in a respectful manner during such activities. (PSVS)

Alternatively, the teacher may wish to use the law of exponents for division of powers as a method of introducing this section.

E.g., demonstrate $x^3 \div x^5 = x^{3-5} = x^{-2}$, and compare

to $$\frac{x^3}{x^5} = \frac{xxx}{xxxxx} = \frac{1}{x^2} = x^{-2} .$$

This topic could be extended to variable bases.

Examples/Activities	**Adaptations**

Examples/Activities

5. $(x-7)\overline{)x^2-51}$

6. $(3x+1)\overline{)6x^2+20x+4}$

7. Find the price of each 'doodad' in a shipment, if $(2x+5)$ doodads are shipped, and the total value of doodads in the shipment is $(16x^2+34x-20)$ dollars.

Evaluate:

1. 2^3
2. 7^4
3. $(-3)^5$
4. $(-3)^6$
5. -3^6
6. 4^{-3}
7. 10^{-5}
8. 10^6
9. 6^{-1}
10. $2^5 \cdot 3^2$
11. $4^{-3}4^4$
12. x^{-4}
13. y^{-3}

14. Rewrite, using only positive exponents:

a) $x^{-3}y^2$

b) $\dfrac{4}{x^5y^{-4}}$

Adaptations

Extend this topic by dividing into polynomials of four or more terms, some of which might not be written in descending order, or some of whose coefficients may be equal to zero.

Students can be asked if they have seen negative exponents elsewhere. If so, how were they used? What if calculators could not use them? What if there was no scientific notation? Why are negative exponents useful in describing small numbers?

E.g.,

$x^2 \cdot x^{-2} = x^2 \cdot 1/x^2 =$?

Students could be instructed to use a calculator to find the result of 2^{-2}, and then be asked to determine the definition.

Concept C: Polynomials and Rational Expressions

Objectives	Instructional Notes
C.4 To simplify variable expressions with integral exponents using the following properties of exponents: product, quotient, power of a product, power of a quotient, negative exponent, and zero exponent.	The basic powers of exponents can be introduced through examples using the `meaning of an exponent'. Students could work in small groups, given an assignment that would require them to expand expressions using the `meaning of an exponent'. The students could discuss their answers and observations with each other, and determine what generalizations could be made. Group answers would be shared with the entire class. (PSVS)
C.5 To determine the non-permissible values for the variable in rational expressions.	Review the definition of a rational number: $(a/b:\ a,b$ å$I, b \neq 0)$. Have students use their calculators to determine what will occur if they divide any number by zero. The teacher can suggest a series of divisors that approach zero (10, 1, .1, .01, .001, .0000001). This may make the students more aware of what is actually occurring, and could also be a quick review of the place-value system. Students might then discuss what happens when you attempt to divide by zero. Extend this to a series of questions where students determine the value of the variable which would result in the denominator being equal to zero. Discuss why this cannot be permitted.
C.6 To simplify rational expressions by factoring.	You may wish to review the process for simplifying numerical fractions, such as $\frac{18}{27} = \frac{6 \cdot 3}{9 \cdot 3} = \frac{2 \cdot (3 \cdot 3)}{3 \cdot (3 \cdot 3)} = \frac{2}{3}$ and extend this process to variable expressions. Stress the goal of finding factors that have a quotient of 1.

Examples/Activities	Adaptations

Examples/Activities

Simplify:

1. $x^5x^{11}=$
2. $6^46^7=$
3. $3x^42x^7=$
4. $4x^2y^36x^5y^4=$
5. $\dfrac{14x^3y^4}{-7xy^2}=$
6. $(-2x^3y^4)^4=$
7. $\dfrac{(3x^2)^3}{(2y)^3}=$
8. $\dfrac{4x^{-3}y^4}{2x^2y^{-3}}=$
9. $12x^0=$
10. $(64x^5y)^0=$

Adaptations

This section could be extended by introducing combinations of various types of questions.

E.g., $\dfrac{(4x^2y)^3(2xy^3)^2}{(3x^{-2})^3}$

Examples/Activities

Determine any non-permissible value(s) of the variable:

1. $\dfrac{4}{x}$
2. $\dfrac{-8}{3x}$
3. $\dfrac{1}{x+2}$
4. $\dfrac{4}{3x+5}$
5. $\dfrac{2x+7}{2x-7}$
6. $\dfrac{4}{(2x+3)(3x-1)}$
7. $\dfrac{4x+3}{x^2+5x+6}$

Adaptations

If a graphic calculator or computer software is available, you might try to graph selected questions, sharing the results with the class. The students can discuss these graphs, decide which characteristics determine the non-permissible values, and what effect these values have on the graph. (NUM)

Examples/Activities

Simplify by factoring:

1. $\dfrac{3x}{12x^2}$
2. $\dfrac{4x+10}{10x+25}$
3. $\dfrac{x+3}{x^2+10x+21}$
4. $\dfrac{2x^2+5x+2}{4x^2+4x+1}$

Note: Non-permissible values could be identified.

Adaptations

This section could be extended by using combinations of factoring in the numerator, denominator, or both.

Concept C: Polynomials and Rational Expressions

Objectives	Instructional Notes
C.7 To multiply and divide rational expressions.	A review of the procedure for multiplying and dividing numerical fractions would establish a framework for the student. Examples of multiplying and dividing variable expressions could then be introduced in a similar manner. (Non-permissible values could be identified by the students.)
C.8 To add and subtract rational expressions involving like and unlike **monomial** denominators.	Review the basic procedures for the addition and subtraction of numerical fractions. Stress that addition and subtraction require equal denominators before any operation can be done with the numerators. A brief discussion of the Lowest Common Multiple, or L.C.M., may be useful.

Examples/Activities

Simplify:

1. $\frac{4x^2y}{3y} \cdot \frac{2y}{3x^2}$

2. $\frac{7x^3y}{2xy} \div \frac{14xy}{3z}$

3. $\frac{x+3}{x-2} \cdot \frac{x-4}{2x+3}$

4. $\frac{x^2+5x-14}{x^2-9} \div \frac{x+7}{x-3}$

Perform the indicated operation:

1. $\frac{6}{5x} + \frac{3}{5x}$

2. $\frac{4}{y} - \frac{3}{y}$

3. $\frac{6x}{2y} - \frac{3x}{2y}$

4. $\frac{2x}{z} + \frac{5}{z}$

5. $\frac{6x+1}{y} - \frac{3x-4}{y}$

6. $\frac{2x+3}{3y} + \frac{5x}{4y}$

7. $\frac{4x-1}{y} - \frac{2x-3}{3y}$

Adaptations

This topic could be extended by multiplying three or more rational expressions, by introducing combinations of multiplication and division, or by using trinomial denominators.

This topic could be extended by adding or subtracting three or more rational expressions, inclusion of parentheses, or by using trinomial (and binomial) denominators.

Concept D: Quadratic Functions

Foundational Objectives

- ! To be aware that the graph of an equation in two variables, where only one variable is of degree two, is a parabola. (10 04 01). Supported by learning objective 1.

- ! To draw graphs of equations representing parabolas. (10 04 02). Supported by learning objectives 2 and 3.

- ! To demonstrate an ability to interpret an equation of the form $y = a(x-p)^2 + q$, as to the effect the values of a, p, and q have on the graph. (10 04 03). Supported by learning objectives 4 and 5.

Objectives	Instructional Notes
D.1 To define a quadratic function.	Review the definition of function, and provide some examples of linear functions, as done in Mathematics 10. The meaning of 'quadratic' could then be introduced, and a quadratic function be defined. Student could be given practice determining whether given expressions would represent quadratic functions.
D.2 To identify, graph, and determine the properties of quadratic functions of the following forms: $f(x) = ax^2$ $f(x) = x^2 + q$ $f(x) = (x-p)^2$ $f(x) = a(x-p)^2 + q$	Students could work in small groups, with each group responsible for the plotting of a series of functions. Each member of the group could plot one or two of the functions. The members of the group could compare their results to determine what effect each constant or coefficient has on the graph of the function. Results from each group could be shared with the entire class. Alternatively, graphic calculators or computer software, if available, could be used to generate the graphs quickly.

Examples/Activities

Students could be arranged in small groups, with each group given a set of expressions, and be asked to determine which are quadratic functions, and which are not. They should be asked to provide a reason for their grouping choices. Each group could report its decisions to the whole class. (CCT)

The class could then determine those features that define quadratic functions.

Group 1 may be given the following questions to plot, and asked to determine the effect of the coefficient `a':

$$f(x)=x^2 \qquad f(x)=2x^2$$

$$f(x)=5x^2 \qquad f(x)=\frac{1x^2}{2}$$

$$f(x)=\frac{1x^2}{4} \qquad f(x)=-x^2$$

$$f(x)=-2x^2 \qquad f(x)=\frac{-1x^2}{3}$$

Group 2 might be asked to do the same for the series:

$$f(x)=x^2+2 \qquad f(x)=x^2+5$$

$$f(x)=x^2-3 \qquad f(x)=3x^2+2$$

$$f(x)=3x^2+5 \qquad f(x)=-2x^2-3$$

Similar series could be designed for the type $f(x)=(x-p)^2$.

For $f(x) = a(x-p)^2 + q$, results of the preceding types could be summarized, properties of each type of function analyzed, and further practice assigned.

Adaptations

Students could be asked to generate equations for specific situations.

For example: given that the graph of an unknown quadratic function has a y-intercept of 4, and has its vertex at (-1,0), what is the value of `a' in $f(x) = a(x-p)^2 + q$?

Students could be given well-labelled graphs representing quadratic functions. From the graphs, they would be expected to identify the key features of the graph.

Concept D: Quadratic Functions

Objectives	Instructional Notes
D.3 To determine the domain and range from the graph of a quadratic function.	Present graphs of several quadratic functions to the class. Review the meaning of domain and examine the graphs to determine the domain of each. Have students note any patterns or similarities that exist. Discuss the range in a similar manner. (NUM)
D.4 To analyze the graphs of quadratic functions that depict real-world situations.	Students may generate cases where these functions exist in the real-world, or the teacher may provide examples. Some examples might be, a 'fly ball' in baseball, other trajectories, satellite dishes, reflectors for headlights, a ski jump, and others. (TL) A reminder of the properties of a quadratic function may help.
D.5 To solve problems involving the graphs of quadratic functions that depict real-world situations.	Stress that the student must determine the quadratic function, and graph it. Only when the graph is complete can the student analyze it to determine its key features.

Examples/Activities	Adaptations
Students could be instructed to refer to graphs from section **D.2**, and be asked to note the domain and range of each. Information from real-world situations could be plotted, and students asked to determine domain and range, and discuss possible consequences.	Students could be asked to determine the domain and range of some non-functions of the type $x = a(y-k)^2$, and to determine if any patterns exist.
Students could be arranged in small groups, given several well-labelled graphs, and instructed to determine the characteristics of each, noting the key features as well. They would then report their conclusions to the class, providing reasons for their choices. (CCT)	Students could be asked to generate their own graphs, depicting real-world situations and have the other students analyze these to determine the important aspects of the function. Math labs could be used as well.
Students can work in small groups to plot the graphs needed for these equations. Then, they can decide which are the important features of their graphs, and report to the class. Alternatively, graphic calculators or software could be used.	

Concept E: Quadratic Equations

Foundational Objectives

! To demonstrate the ability to solve quadratic equations by factoring, and by taking the square root of both sides of an equation. (10 05 01). Supported by learning objectives 1, 2, and 3.

! To demonstrate the ability to solve equations containing one radical. (10 05 02). Supported by learning objectives 4 and 5.

Objectives	Instructional Notes
E.1 To solve quadratic equations by: a) factoring, and b) by taking the square roots of both sides of an equation.	Review the meaning of `solving' an equation, and also review the multiplicative property of zero. (ie. ab = 0 if and only if a = 0, b = 0, or a and b both equal zero.) The teacher might have students practice setting equations equal to zero, prior to actually finding solutions of such equations. When using the square root property, stress that there are two possible answers in most cases. For both types of solutions, emphasize the fact that both answers should be checked to see if they actually represent the solution to the equation. This section is largely a review of the Pythagorean Theorem as taught in Math 10.
E.2 To calculate the exact value of the length of a side of a right triangle using the Pythagorean Theorem.	The major difference is the introduction of exact values (radicals), as opposed to giving answers to two decimal places.
E.3 To solve word problems involving quadratic equations.	Caution should be exercised in choosing these problems, as they should be solvable using factoring or the square root property. A variety of problems should be chosen, from several different areas: business, commerce, engineering, and geometry as examples.

Examples/Activities	Adaptations

Students could be asked to graph several quadratic equations of these types, and be instructed to note the points at which the graph crosses the x-axis. A graphic calculator or software might be useful in this exercise.

Solve:

1. $x(x-3)=0$
2. $x^2+5x=0$
3. $(x-4)(x+2)=0$
4. $x^2-3x-10=0$
5. $x^2+10=7x$
6. $3x^2=10x-3$
7. $x^2=9$
8. $(x+2)^2=25$
9. $(3x-1)^2=9$
10. $(4x-3)^2-0.25=0$

Students might be asked what occurs in a situation where no solution can be found. They could be instructed to graph this type of function to obtain a geometric interpretation.

Eg. $0=x^2+4$ could be graphed as $f(x)=x^2+4$.

Determine the value of the variable in each case.

1. 2. 3.

4. 5.

A discussion on the historical significance of this topic might be appropriate.

Alternatively, students might be instructed to do some research about the Pythagorean School, Pythagoras, or Eratosthenes. (IL)

Students could work in small groups. Some members of the group could be asked to solve a problem algebraically, others could solve the same problem graphically, and still others could be instructed to check the answers to see if they meet the conditions of the problem.

These roles could be rotated every question, so that each member of each group has an opportunity to participate in all aspects of problem solving.

Students could be asked to research areas where these types of situations occur in real life and to share these examples with the class.
Note: Student examples may not have factorable solutions.

Concept E: Quadratic Equations

Objectives	Instructional Notes
E.4 To solve and verify radical equations containing one radicand.	Students should be instructed to check their answers against the criteria outlined in the question, as only one of the answers may represent the correct solution. Have students review the procedure for squaring radicals. For this topic, stress that the radicand should be isolated before squaring both sides. A check of the solution is both required and necessary.
E.5 To solve problems that involve equations which contain radicals.	Students should be instructed to check their answers carefully, to determine if the answer meets the criteria outlined in the problem.

Examples/Activities

Solve:

1. $\sqrt{x} = 2$ 2. $\sqrt{x} = -3$

Have students do both types of equations. Discuss permissible and non-permissible values.

3. $\sqrt{x+5}=7$ 4. $3\sqrt{2x+3}=12$ 5. $6\sqrt{3x-5}-7=11$

Adaptations

This topic can be extended by using questions involving a radicand, and another term using the same variable.

For example: $x=\sqrt{5x+6}+6$

Concept F: Probability

Foundational Objective

! To appreciate the role of probability in understanding everyday situations. (10 06 01). Supported by the following learning objectives. (NUM)

Objectives	Instructional Notes
F.1 To list the sample space and events for a random experiment.	Introduce the concept of probability by having students brainstorm for possible areas where they have heard probability used; e.g., weather, surveys, lotteries, games, genetics.
F.2 To calculate the experimental probability of simple events by performing repeated experiments.	Students learn by becoming involved in activities. Games and experiments allow the students to immerse themselves in the physical aspects of probability. Following up with a discussion that involves questions and answers helps to elicit the definition of the underlying concepts of probability. Possible questions might include: Do the results agree with what you thought would happen? Would you see the same results by repeating the experiment? Would the results be different if the experiment were repeated with a larger sample? Do the results help us to make practical decisions? If A is the event desired, then $P(A) = \frac{\text{number of favourable outcomes}}{\text{number of experimental trials}}$ Students should be aware that experimental data only approximate theoretical outcomes.
F.3 To calculate the theoretical probability of an event and the probability of its complement.	Have the students determine the theoretical probability of the previous experiments and compare these results to the experimental probability. If you increase the number of trials in the experiments, how would the experimental probability compare with the theoretical probability? $P(A) = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$ Use tree diagrams to show the sample space and the probability of an event and its complement. Note that the $P(A) + P(\overline{A}) = 1$.

Examples/Activities	**Adaptations**
Conduct various experiments and have the students record the sample space for each; e.g., tossing coins, dice, and thumbtacks.	
In pairs, and using one coin, students play heads versus tails for 50 tosses. They determine the experimental probability for tossing a head or tail. The total number of heads and tails combine to give a better value of the probability. Die can be tossed to determine the probability of showing a 2, 5, or an even number. Spinners may also be used to determine the probability of a certain number or colour.	Research and examine the practical uses of empirical probability concerning weather, death, fire, accidents, crime rates, and consumer preference for various brands or styles of products. (NUM) Simulate experimental probability on the computer using the BASIC command RND(x).
Use a standard deck of playing cards and count out 20 cards. Do not look at them. Perform an experiment to estimate the probability of drawing a club, a diamond, a heart, and a spade from the deck of 20 cards. Make 20 draws from the deck, replacing the card each time, and recording its suit. Determine the estimate of the probability of drawing a heart, diamond, spade, and club. Look at the 20 cards and count the number of cards in each suit. Determine the theoretical probability of obtaining a heart, diamond, spade, and club. How does the theoretical probability compare to the experimental probability?	Students can investigate the theoretical probability of specific hereditary characteristics.

A die is rolled. What is the probability of rolling an even number? What is the probability of its complement?

(1, 2, 3, 4, 5, 6)

Event A	*Event* $\overline{A}$
(2, 4, 6)	(1, 3, 5)

$P(A) = 3/6$
$= 1/2$

$P(\overline{A}) = 3/6$
$= 1/2$

Concept G: Angles and Polygons

Note: Computer software such as GeoDraw (IBM) or Geometric Supposer (Triangles) or (Circles) could be used throughout this Geometry unit.

Foundational Objectives

- ! To develop the ability to identify pairs of congruent triangles and to employ the congruence postulates SSS, SAS, ASA, AAS, or HL in guided proofs showing such congruences. (10 07 01). Supported by learning objectives 1 to 5.
- ! To demonstrate the ability to apply the concepts of similar polygons and scale factors to determine the surface area and/or volume of similar polygons or solids. (10 07 02). Supported by learning objectives 8 to 15.
- ! To provide a reasonable explanation for congruences of pairs of triangles, or for corresponding parts of congruent triangles (10 07 03). Supported by learning objectives 6 and 7.

Objectives	Instructional Notes
G.1 To informally and formally construct congruent angles and congruent triangles.	Informal construction of congruent angles and congruent triangles can be done using any of several methods; paper folding, mira, tracing, or measurement with ruler and protractor. Formal construction of these figures requires a Euclidean approach, with the utilization of a straightedge and compass, or some alternatives that have been recently marketed.
G.2 To determine the properties of congruent triangles.	Students should be given a series of diagrams of congruent triangles, and then asked to determine what features the triangles have in common. (CCT) The students should reach the conclusion that all three sides and all three angles of the congruent triangles are equal (congruent).
G.3 To identify and state corresponding parts of congruent triangles.	Given two congruent triangles, stress that there are six pairs of congruent parts of these triangles. Have the students list these six correspondences. Students should also be made aware of the technique used for 'marking' these congruent correspondences on the diagrams. Students should also be able to list these congruent correspondences when given a congruence statement.

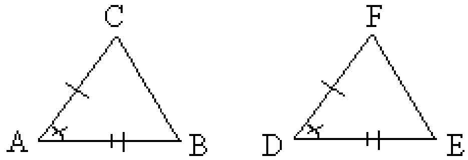
Examples/Activities

A discussion of informal techniques can bring out some of the deficiencies of these methods. Students might also discuss where informal methods of construction would be suitable and where more formal methods would be appropriate.

Practice should include several different types of angles. Triangles should be constructed given different sets of correspondences for each question.

Students can discuss this topic, either as an entire class, or in small groups. The conclusion should be reached through a summary of class comments.

1. Given the triangles as marked, state the corresponding parts and the triangle congruence.



2. Given the statement $\triangle JKL \cong \triangle ARD$, draw the triangles and list all pairs of corresponding parts.

Adaptations

Students could be asked to note the step-by-step procedure used in formal constructions. This might be written in the traditional manner, in point form, or as a flowchart of steps. (COM)

The students could then have classmates follow their description to determine if their directions were correctly communicated.

Students could be given a list of three or more congruent correspondences, and then asked to: a) draw the congruent triangles, and b) determine if such triangles exist given the stated congruences.

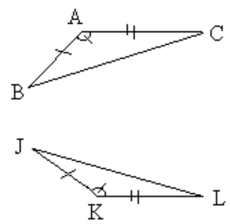
Concept G: Angles and Polygons

Objectives	Instructional Notes
G.4 To determine whether triangles are congruent by SSS, SAS, ASA, AAS, or HL.	The use of a series of diagrams is suggested.
G.5 To prove that two triangles are congruent by supplying the statements and reasons in a guided deductive proof.	Care should be exercised in choosing the first examples and questions. Most of the proofs in this section should be relatively simple. The concept of 'proof' is a relatively new one to these students, and may require some introductory discussion of supplying "reasons why" for a statement. (CCT)

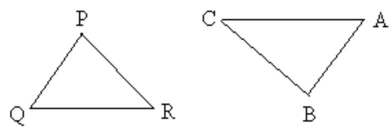
Examples/Activities

Students could work on a series of diagrammatic exercises of the following types.

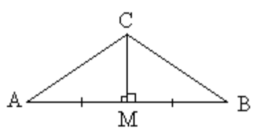
1.



2. $\overline{AB} \cong \overline{PI}, \angle A \cong \angle Q, \angle B \cong \angle P$ Given

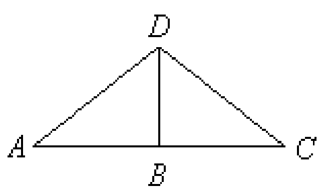


3.



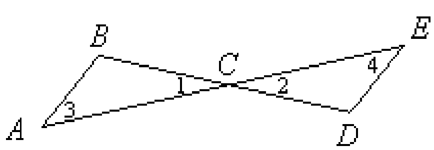
1. Given: $\overline{AB} \cong \overline{BC}$, $\overline{AD} \cong \overline{CD}$

Prove: $\triangle ABD \cong \triangle CBD$



2. Given: $\angle 3 \cong \angle 4$, $\overline{AC} \cong \overline{CE}$

Prove: $\triangle ABC \cong \triangle EDC$



Students could work in small groups to identify similar polygons in a given assignment.

Adaptations

This topic could be extended by supplying only written information. The students would be expected to provide a diagram, and the correct congruence. For example:

Given: $\angle A \cong \angle D$
$\overline{AB} \cong \overline{DE}$
$\overline{CA} \cong \overline{FD}$

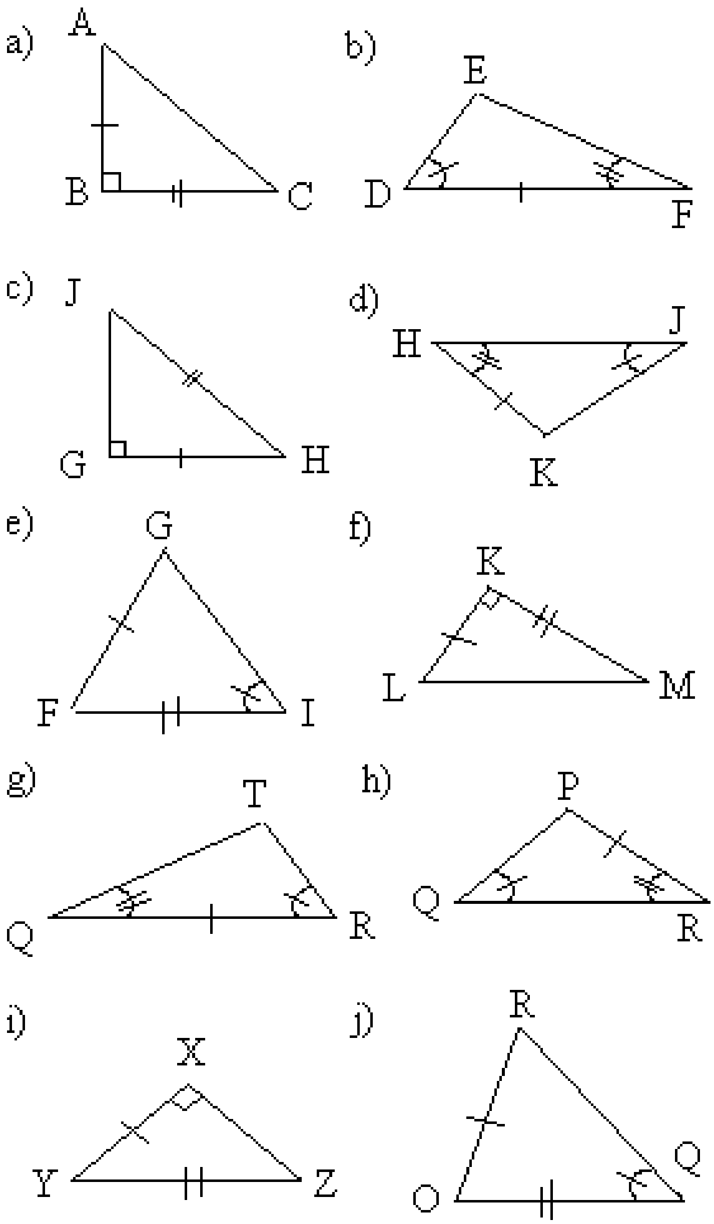
More difficult proofs can be introduced for students needing a challenge.

Modifications can be made by providing the 'statements' and asking students to supply the 'reasons' in a given proof.

Objectives	Instructional Notes
G.6 To prove triangles congruent by SSS, SAS, AAS, ASA, or HL in a two-column deductive proof or paragraph form.	To introduce this section, instruct students to construct triangles given the measures of two sides and the included angles (SAS). Have them compare their triangle to others in their group. They should recognize that these are congruent. Similar instructions may be given for the other congruence postulates. Also, have them construct triangles when given only the three angles, and compare with other students (also SSA). This should review the necessary characteristics for triangle congruence. Also review the standard nomenclature for triangles. Provide the students with a pair of triangles such that congruent parts are indicated (SAS). Ask the students if these are congruent. Instruct them to justify the congruence by listing the congruent parts. Once these are listed, ask them for their conclusion (triangles are congruent). Then ask them to complete the second column, justifying each step. Distribute a set of cards with pairs of triangles to each group, and have them do similar proofs. Groups can exchange cards when they have completed their work.

Examples/Activities

1. Identify the pairs of triangles that are congruent.



2. For each pair in question 1 that is congruent, state the postulates that are congruent. List the parts of the triangles that are congruent, and justify each statement. Formalize these proofs.

Adaptations

Students can also be given the information on triangles written in terms of triangle correspondences.

Given:

$$\overline{AB}\cong\overline{DE},\ \overline{BC}\cong\overline{EF},\ \angle B\cong\angle E$$

Prove:

ÄABC≅ÄDEF

Then ask them to complete a proof. In the initial stages, students may be instructed to write in paragraph or point form why them believe a certain pair of triangles is congruent. They may use this as a `bridge' to writing formal proofs.

Objectives

G.7 To prove corresponding parts of congruent triangles are congruent.

Instructional Notes

Have students demonstrate their knowledge of the concept of congruent triangles by providing students with a pair of labelled congruent triangles, and instruct them to list all six pairs of congruent parts.

Provide the groups of students with cards having pairs of congruent triangles marked as in the previous objective. Instruct the students to prove the triangles congruent and then to list the other corresponding parts of the triangles that are congruent.

The last stage is to provide cards with pairs of triangles marked so that the students are to prove a specific pair of corresponding parts are congruent.

Stress that naming the triangles using corresponding parts will help in identifying these.

As an example, ask how the triangles in 2b) on the opposite page should be named so that the corresponding parts are named in the same order.

This habit pays dividends in the later study of similar triangles.

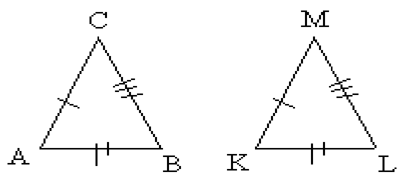
Examples/Activities

1.a) Given

$$\triangle ABC \cong \triangle DEF$$

list all pairs of corresponding congruent parts.

b) Given the diagram as marked below, list all pairs of corresponding congruent parts.



2.a) Given

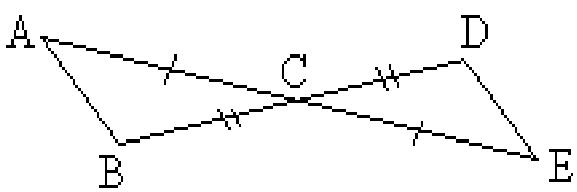
$$\overline{AB} \cong \overline{KL}, \angle A \cong \angle K, \angle B \cong \angle L$$

Prove

$$\triangle ABC \cong \triangle KLM$$

and list all other pairs of corresponding congruent parts.

b) Given the diagram below, prove the triangles congruent, and then list all other pairs of corresponding congruent parts.



3.a) Given that <A and <X are right angles

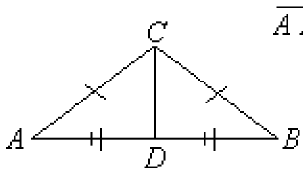
$$\overline{BC} \cong \overline{YZ}, \overline{AC} \cong \overline{XZ}$$

Prove

$$\angle B \cong \angle Y$$

b) Given the diagram below, prove that

$$\overline{AD} \cong \overline{ED}$$



Adaptations

Many different examples of these types of proofs can be found in geometry texts. Try to use a selection of proofs that encourages students to identify the triangles that must be proved congruent before they begin the proof. There are also exercises that require students to prove more than one set of triangles congruent before they reach the final conclusion. Some of these can be expected to be completed by better students, or by groups working together.

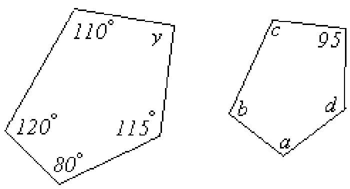
Concept G: Angles and Polygons

Objectives	Instructional Notes
G.8 To identify similar polygons.	Provide, or develop, the definition of similar polygons. Do several examples for the entire class.
G.9 To determine the measure of corresponding angles in two similar polygons.	Diagram examples can be presented to the students. Students may need to be reminded of the definitions used.
G.10 To calculate the scale factor of two similar polygons.	Define the term `scale factor'. Students could be asked to provide examples of "scale factors" at use in the real world; e.g., blueprints, model planes, trains, maps, photographs, etc.
G.11 To calculate the length of a missing side of two similar polygons.	Diagrams can be used to introduce this topic to students. A reminder of the role of scale factor with similar polygons may be useful.

Examples/Activities | **Adaptations**

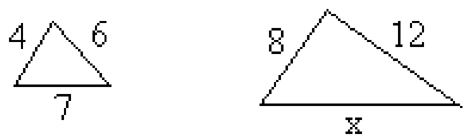
Find the measure of the angles indicated by variables, given the polygons are similar.

1.

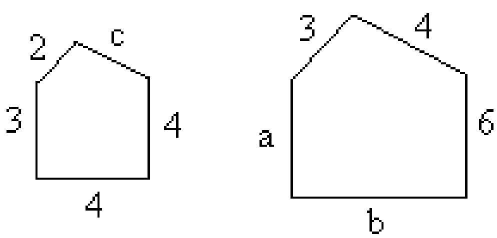


Determine the scale factor of each pair of similar polygons. Determine the value of indicated variables.

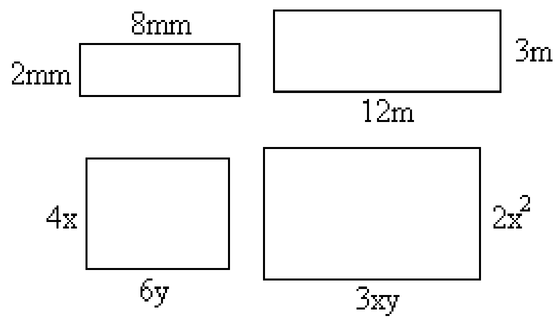
1.



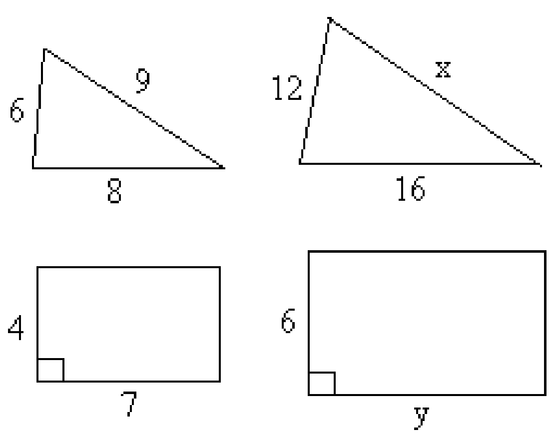
2.



This could be extended by using large scale factors given in different units, or by using variables as scale factors.



Students can be arranged in pairs to complete a set of exercises on this topic cooperatively. They could be instructed to generate a small number of these types of questions for each other.



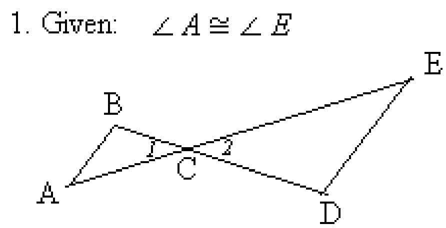
This topic can be extended by using corresponding sides of different units, or scale factors which do not represent common decimal fractions.

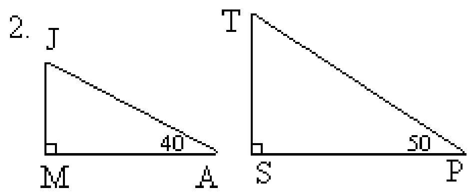
Concept G: Angles and Polygons

Objectives	Instructional Notes
G.12 To show that two triangles are similar by the Angle Angle Similarity Theorem. (Postulate in some resource texts).	Review the definition of similar triangles. Have students find the third angle of a triangle, given the first two. Ask them to determine how many angles they need to know, before all the angles of the triangle can be identified. The Angle Angle Similarity Theorem can be introduced at this point.
G.13 To calculate the length of a missing side in two similar right triangles.	In this section, the activity can be extended to include the Pythagorean Theorem, as well as the scale factor of similar polygons.
G.14 To solve problems involving similar triangles, and other polygons.	Problems from real-world situations may be used. The concept of determining a distance by triangulation should be discussed, as well as the reasons why triangulation is an effective method.
G.15 To determine surface area and volumes of similar polygons or solids.	The relationship of a scale factor to an area or volume should be established. Initial examples could use line segments or sides which have an integral scale factor. Squares and cubes could be used to show the relationships of area and volume to these scale factors. Other types of figures, polygons and solids, could then be introduced.

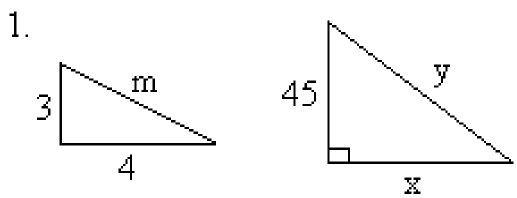
Examples/Activities

Have students identify similar triangles by listing two pairs of congruent corresponding angles in each, and naming the triangles correctly.
E.g., state the similar triangles:





Find the value of the indicated variables:



1. A blueprint of a house includes a scale drawing of a triangular roof truss. On the diagram, the truss' base is 24 cm and the height is 8 cm. The actual truss is to have a base of 7.2 m. What is the actual height of the truss?

1. Two similar polygons are such that their lengths have a ratio of 1:3. What is the ratio of their surface areas, and their volumes?

2. A model airplane is constructed on a scale of 1:50. If the volume the model displaces is 1.5 L, what volume would the real plane displace?

Adaptations

This section could be extended by introducing the altitude drawn to the hypotenuse of a right triangle. This forms several similar right triangles, and uses the geometric means in various contexts.

How much more should Melanie charge for a 30 cm pizza than for a 23 cm pizza? What are some other factors that might be taken into account when determining this price?

Concept H: Circles

Foundational Objective

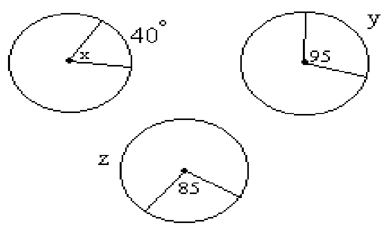
! To develop and display an understanding of certain relationships of the chords, tangents, and arcs of a circle. (10 08 01). Supported by the following learning objectives.

Objectives	Instructional Notes
H.1 To define the measure of a minor arc, and to calculate the measure of a central angle.	The definition of measure of a minor arc should be discussed and various examples presented. Students should be made aware of the relationship between arcs and central angles.
H.2 To determine the relationship that exists between the following: 1. The radius of a circle and a tangent line drawn to it at the point of tangency.	Definitions of new terms should be provided. Students might work in teams to draw models of the statement of the objective and be asked to state their observations and conjectures. Their responses could be checked with a protractor or set square.
2. Two tangents drawn to a circle from the same point.	Students could work in pairs, with each member of their pair drawing a diagram that satisfies the statement of the objective. The students could check each other's diagram. (PSVS)
3. Chords and arcs in the same circle or in congruent circles.	Introduce new terms through definitions, and examples. Students can be instructed to draw diagrams illustrating these on their own, to make the central angles congruent, (either formally or informally), and then to measure the chords. A sampling of various student answers, followed by some discussion, should lead to a logical conclusion. Alternatively, the teacher may use a more direct approach, modelling the situation for the entire class.
4. A diameter and a chord bisected by the diameter.	Students could be asked to diagram this statement and then be instructed to find the measures of the segments of the chord, and of the central angles. Various student responses, followed by a brief discussion, should end with a logical conclusion. Alternatively, the teacher could model the situation for the entire class.

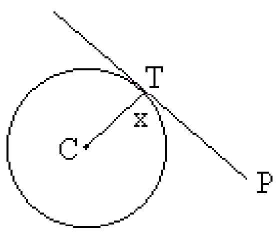
Examples/Activities

Find the value of the indicated variable:

1.



A few numerical examples might be done, simply to reinforce the objective.
E.g., find the value of x.



Once the diagrams are drawn, students could be asked to measure the two tangents drawn from the same point, and to discuss the result. A second diagram could be drawn by each, and the procedure repeated, to check possible answers.

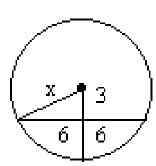
1. Students draw several pairs of congruent circles, choose different chord lengths for each pair, and determine the central angles of each pair.

2. Exercise #1 could be repeated with students choosing different central angles for each pair and measuring the lengths of the chords in each pair.

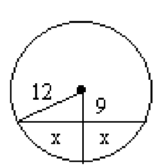
3. Exercises #1 and #2 can be repeated using a single circle.

Find the indicated missing value:

1.



2.



Adaptations

This concept is typically dealt with by definition. Stress the importance of this definition, as it forms the basis by which the other relationships are studied.

A formal proof could be used to challenge students.

A formal proof could be used to challenge students.

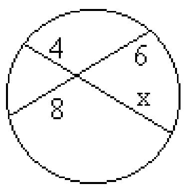
Some students could be expected to provide a formal proof.

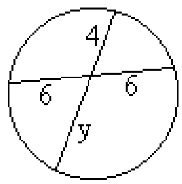
Objectives	Instructional Notes
5. Two chords that intersect in a circle.	Students could be directed to draw a circle with two intersecting chords, to measure the segments of the chords, and to multiply together the segments of each chord. Various student responses could be taken. Alternatively, the teacher could employ a direct inquiry approach utilizing similar triangles.
H.3 To solve problems based on the relationships stated in G.2.	Students may need to be reminded that it is necessary to decide which relationship from G.2 is described in the problem.

Examples/Activities

1. Students could be instructed to exchange diagrams with a partner. They could measure the segments on these diagrams and carry out the multiplication.

2. Find the missing values:

a) 

b) 

Students could be directed to generate their own word problems to exchange with a partner:

1. Inscribe a regular pentagon in a circle. What is the measure of each angle?

2. A quadrilateral is inscribed in a circle. One angle is 108°. What is the measure of the opposite angle?

Adaptations

Students could provide a proof of this statement using their knowledge of similar triangles, ratio, and proportion.

Teacher Notes

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The following materials were used as background during the preparation of this document:

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Appendices

Appendix A

Instructional Strategies and Methods

The following include precis of some methods described in *Instructional Approaches: A Framework for Professional Practice* (1991). Refer to this document for a more detailed discussion of these methods and the "families" of strategies to which they belong.

Cooperative Learning

This is an approach where students work together to complete a task or project which is often based on their particular group's strengths and interests. Students engage in brainstorming, reflective discussion, mutual decision making, or conducting research. The purpose of using cooperative learning groups is to minimize competitiveness and feelings of low self-esteem and to increase students' respect for and understanding of each others' unique abilities, interests and needs. (PSVS)

Concept Attainment

Concept attainment focuses on understanding what characteristics or attributes may be useful for distinguishing between members and nonmembers of a group or class. Clues are supplied by the teacher from which students are to determine the identity of concepts. The five key elements necessary to define a concept are: names, examples, attributes, attribute values, and rules. (CCT)

Lecture

The learning environment during lecture is task centred, teacher directed, and highly structured. In general, this strategy involves the teacher explaining a new concept to the large group, the testing of student-understanding by controlled practice, continued guided practice under the teacher's supervision, and finally independent practice by the student.

Problem Solving

Problem solving is the process of accepting a challenge and striving to resolve it. It allows students to become skillful in selecting and identifying relevant conditions and concepts, searching for appropriate generalizations, formulating plans, and employing acquired skills. The process of problem solving involves three phases: understanding the problem, devising and carrying out the plan, and looking back. (CCT)

Learning Centres

Learning centres may include examples of print and non-print materials, types of audio-visual equipment, and programmed instruction. The purpose is to provide students with differentiated learning experiences in the form of individual or group activities. Students may be directed by the teacher or may be given the opportunity to select, manage, and evaluate the experiences around which the centre was designed. (IL)

Drill and Practice

Drill and practice can be of many types; visual, manipulative, oral, written, or any combination of these. To be of most value it must always be accompanied with good mental processes. Drill and practice should follow the developmental and discovery stages of learning and be used to reinforce and extend basic learning.

Compare and Contrast

This strategy develops the students' ability to collect, organize, and remember information and then helps them apply that information for new learning. This method consists of three phases: the description phase, comparison phase, and application phase. By keeping the students actively and thoughtfully involved in collecting and processing information, the compare and contrast method helps them develop the ability to become independent learners. (CCT)

Role Playing

Role playing allows students to deal with problems through action: through identifying the problem, acting it out, and discussing it. The essence of role playing is the involvement of participants and observers in a real problem situation and the desire for resolution and understanding that this involvement generates.

Games

A well-designed game will partly teach itself. Simulations enable students to learn first-hand from the simulated experiences built into the game rather than from teacher's explanations. However, it is still important that a teacher raise the students' consciousness about the concepts and principles involved in the game, because the students may not always be aware of what they are learning and experiencing.

Projects

Projects are used for independent study and should supplement or enrich the basic lesson, once the skills or concepts have been learned. They foster the development of individual student initiative, self-reliance, and self-improvement. The role of the teacher is one of resource person rather than a presenter. (IL)

Computer-Assisted Instruction

This independent study method promotes learning through interactive demonstration, drill and practice, tutorial, simulation, educational games, and programming as problem solving. Teachers must carefully assess the software under consideration so that it meets the desired educational goals.

Tutorial Groups

Tutorial groups are set up to help students who need remediation or additional practice, or for students who can benefit from enrichment. A tutorial group is usually led by the teacher. Tutorial groups provide for greater attention to individual needs and allow students to participate more actively. Peer tutoring occurs when a student (the tutor) is assigned to help other students (the learners). The roles played by teacher, tutor, and learner must be explained and expectations for behaviour must be outlined.

Reflective Discussion

Effective application of reflective discussion requires a range of skills in conducting and concluding a discussion. The categories of skills include: interpersonal exchanges, motivating, questioning, and reinforcement. The discussions are based on material familiar to the students. The teacher should set the stage, stress that opinions must be supported, and then ensure that the terms and concepts needed are understood. Discussion should conclude with consensus, a solution, insights gained, or a summary, that is preferably provided by the students. (COM)

Guided Inquiry

This approach focuses on providing opportunities for students to experience and acquire processes through which they can gather information about the world. This requires a high level of interaction among the learner, the teacher, the resources, the content, and the learning environment. The teacher has to strike a balance between telling and helping the learners to discover for themselves.

Further information and examples of instructional strategies and methods can also be found in the *Instructional Strategies Series* of booklets published by SIDRU/SPDU.

Appendix B

Credit

The following is an excerpt taken from *The Interim Report of the Credit Education Project* (Consumers' Association of Canada, Saskatchewan Branch, 1993).

Background

Consumer credit, when used properly, has many benefits:

! it permits people to purchase high cost items such as a house, car, or large appliance without having to pay the full price up front;
! it increases consumers' freedom of choice and may result in savings;
! it provides a source of money for unexpected needs and emergencies; and,
! it can provide a money management regime and a budget system.

Credit can also be a big problem for many people if they can't make the distinction between living with credit and living with debt.

! Individuals, families, and communities are experiencing economic pressures. These are due to the troubled farm economy, wages that do not bring families above the poverty line; job insecurity and the growing need for two incomes to support a family.
! There was a rise in total consumer debt in Canada from $46 billion in 1982 to $92.6 billion in 1988. Only a small portion of this increase may be attributed to population growth.
! Today in Canada, there is a clear indication that many consumers do not know how to handle credit. In the summer of 1990, the Conference Board of Canada reported that consumer debt is increasing as a proportion of personal disposable income. The total household debt of the average Canadian household (mortgage plus other credit costs) now eats up about 79% of personal disposable income.
! In 1991 there were 1607 consumer bankruptcies in Saskatchewan with $58,693,183 in assets collected by trustees to offset $72,570,342 in liabilities resulting in a deficiency of $13,877,159. Of these, 67 bankrupts were from Moose Jaw, 61 from Prince Albert, 448 from Regina, 559 from Saskatoon, and 472 from other areas of the province. Many are tied to farm bankruptcies.
! Approximately 10% of consumer bankrupts are repeating the process for a second, third, or even fourth time. The Clare report on *Repeat Bankruptcies of Consumer Debtors* observes that within their family of origin, many bankrupts lacked the benefit of an effective role model, and consequently did not learn essential financial skills.

"Virtually all consumer bankrupts do not:
! know how to handle money or receive reasonable value for the purchases they make;
! understand the concept of credit -- how it works and the responsibilities that go with it;
! as a basic minimum, try to balance income against expenses by keeping an informal monthly budget; and,
! save small amounts as a hedge against the special expenses, events or emergencies that everyone experiences from time to time."

Many of these debtors could have avoided their financial problems if they had learned and applied sound money and credit management skills.

An indispensable aspect of a properly functioning consumer credit market is an informed consuming public. A significant portion of the credit-consuming public in Saskatchewan is not well enough informed to make the proper choices when using consumer credit. The result is personal hardship for the credit consumers involved and increased cost incurred by public and private agencies charged with the responsibility to assist over-committed debtors. Further, if consumer credit is granted and consumed unwisely, the cost of credit to all consumers will inevitably be higher. A credit grantor's losses will be built into the cost of all credit advanced. In addition, there is little hope that competitive market forces can be a significant factor in regulating the consumer credit market so long as consumers are unable to make informed decisions when using credit. The lack of effective competition means that the cost to the public of using consumer credit is higher than it need be.

Appendix C

Reasons for Proofs

1. Given

2. Definition of perpendicular lines
   ! Two lines that meet or intersect to form right angles.

3. Definition of right angle
   ! An angle with measure 90

4. Definition of angles bisector
   ! The bisector of <ABC is a ray BD in the interior of <ABC such that <ABD = <DBC.

5. Definition of segment bisector
   ! A line, segment, ray, or plane that intersects the segment at its midpoint.

6. Definition of perpendicular bisector of a segment
   ! A line, ray, or segment that is perpendicular to the segment at its midpoint.

7. Definition of altitude of a triangle
   ! The perpendicular segment from a vertex to the line containing the opposite side.

8. Definition of median of a triangle
   ! A segment from a vertex to the midpoint of the opposite side.

9. Definition of midpoint of a segment
   ! The point that divides the segment into two congruent segments.

10. Definition of complementary angles
    ! Two angles whose measures have the sum 90.

11. Definition of supplementary angles
    ! Two angles whose measures have the sum 180.

12. Properties of Equality
    a. Addition Property
       If a = b and c = d, then a + c = b + d
    b. Subtraction Property
       If a = b and c = d, then a - c = b - d
    c. Multiplication Property
       If a = b, then ca = cb
    d. Division Property
       If a = b and c = 0, then $\frac{a}{c} = \frac{b}{c}$
    e. Reflexive Property
       a = a
    f. Symmetric Property
       If a = b, then b = a
    g. Transitive Property
       If a = b and b = c, then a = c
    h. Substitution Property
       If a = b, then either a or b may be substituted for the other in any equation or inequality.

13. The sum of the measures of the angles of a triangle is 180°.

14. Vertically opposite angles are congruent.

15. Parallel Lines
    a. If two parallel lines are cut by a transversal, then the alternate interior angles are congruent.
    b. If two parallel lines are cut by a transversal, then the corresponding angles are congruent.
    c. If two parallel lines are cut by a transversal, then the same side interior angles are supplementary.

16. Congruence postulates

    a. SSS
    b. SAS
    c. ASA
    d. AAS
    e. HL